Heterogeneous Expectations and the Business Cycle*

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We analyze the empirical relevance of heterogeneous expectations and central bank credibility in a canonical New Keynesian model subject to the effective lower bound (ELB). Agents switch between an anchored rational expectations (RE) and an adaptive learning forecast rule, where the latter may result in a de-anchoring of inflation expectations. We estimate the model for the U.S. economy using aggregate macrodata and survey data on inflation expectations. We use the estimated model to examine the interaction between the risk of deflationary spirals and central bank credibility at the ELB. A loss of central bank credibility increases the probability of deflationary spirals, highlighting the importance of keeping inflation expectations anchored during periods of uncertainty.

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1. Introduction

Following the Global Financial Crisis (GFC) of 2007–08, leading central banks around the globe cut their nominal interest rates to near-zero levels and encountered the effective lower bound (ELB) constraint on their rates. This has led to an increased volume of research about the relevance and impact of the constraint on the economy. In the aftermath of the Great Recession, central banks have increasingly relied on communication policies in the form of forward guidance and signaling, which have become an important

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pillar of many central banks' unconventional policy toolkit. Given the rise in the frequency and intensity of communication-based tools by central banks, an important and closely related issue is central bank credibility and its interaction with the business cycle dynamics at the ELB.

The macroeconomic literature often examines the impact of ELB on business cycles using the rational expectations (RE) approach. In a standard model, this approach assumes that agents in the economy have complete knowledge of the central bank's objective function and trust that future policy actions will align with that objective. Consequently, there is little room to study the relevance of endogenous central bank credibility in a model with rational expectations. In this paper we relax the full rationality assumption and propose a heterogeneous expectation model with limited information, where agents are allowed to switch between two types of forecasting rules.

As our starting point, we use the canonical three-equation hybrid New Keynesian model, subject to the ELB constraint on nominal interest rates. We introduce heterogeneous expectations to this framework, where agents are allowed to switch between an anchored pseudo-rational expectation model and an adaptive learning model where expectations may become de-anchored if certain conditions are met. The switching mechanism between these two types of expectations is endogenous in the model, where the relative agent shares using each type of forecasting rule depend on their past predictive performance.

A key novelty of our model is that when a high proportion of adaptive learners is combined with the ELB constraint, the economy loses its stability. In these cases, a rising share of adaptive learners corresponds to a loss of trust in the central bank's ability to circumvent the ELB constraint through unconventional monetary policy measures. Consequently, more agents abandon the rational expectations rule and switch to adaptive learning. The presence of more adaptive learners weakens the feedback channel from the central bank's desired interest rate path (shadow rate) to inflation and output gap, which intensifies deflationary pressures. Combined with the ELB, this leads to a higher real interest rate and depresses aggregate demand. Adverse shocks can trigger deflationary spirals under such circumstances if the share of adaptive learners exceeds a critical threshold, where expectations become de-anchored on the downside and the central bank is unable to combat ever-falling inflation and output gap due to the ELB constraint.

We estimate the model for the United States using historical data on consumer price index (CPI) inflation, federal funds rate, and gross domestic product (GDP) as well as short-term (one-quarter) and long-term (10-year) inflation expectations. For inflation expectations, we utilize a novel index of the term structure of inflation expectations, ATSIX, proposed in Aruoba (2020) and regularly published by the Federal Reserve Bank of Philadelphia.[1] To account for the ELB constraint in our sample, we use a regime-switching approximation in the estimation procedure. With the introduction of adaptive learning and time-varying shares of agents, the model is characterized by a conditionally linear structure. We combine this with the standard filtering algorithm in regime-switching literature à la Kim and Nelson (1999) and reformulate the model in state space form with time-varying parameters, which allows us to estimate the structural parameters of the model with standard Bayesian Markov chain Monte Carlo (MCMC) methods. We use the estimated model to pin down the conditions that are needed for deflationary spirals to occur, as well as to assess the likelihood of encountering such scenarios.

The paper is closely related to Özden and Wouters (2021), where a medium-scale dynamic stochastic general equilibrium (DSGE) model is estimated with various representative agent learning rules to examine their fitness before and after the Great Recession. In this study we make use of the same estimation methodology for adaptive learning models developed in that paper. There are two key features that distinguish our current analysis: First, we allow for heterogeneity of expectations, which is a crucial channel both for fitting the data and to study endogenous central bank credibility. Second, we use inflation expectations to estimate the model, which allows for a more robust identification of the parameters related to learning and heterogeneity.

It is important to distinguish the deflationary spiral channel studied in this paper from those that emerge in RE models. As shown in Bianchi, Melosi, and Rottner (2021), deflationary spirals can also

[1]The details of the index can be found at https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/atsix.

arise in a fully rational setup. When low long-run interest rates are combined with agents' expectations about future ELB regimes, a deflationary bias can occur even when the ELB is not binding. If the deflationary bias becomes excessive, the RE equilibrium loses its determinacy and deflationary spirals occur. This highlights the *monetary policy rule* channel of deflationary spirals, where the possibility of hitting the ELB in a future period renders symmetric monetary policy rules sub-optimal. The central bank can mitigate the risk of deflationary spirals by implementing an asymmetric rule instead, whereby its response to inflation above target is slower than its response to inflation below target. This reduces the risk of encountering the ELB regime in the future, which in turn reduces the risk of deflationary spirals. In contrast, our analysis focuses on the *central bank credibility* channel, where deflationary spirals can be mitigated by anchoring expectations at the desired equilibrium. This underscores the importance of effective central bank communication policies to minimize the associated risk.

The key results of the paper are as follows: (i) The heterogeneous expectation model fits the data better than a pure RE or pure adaptive learning model. The model performs particularly well in terms of generating realistic inflation expectation dynamics. (ii) The estimated shares of rational and adaptive agents during the ELB regime 2009–15 are close to 50 percent for the United States, suggesting that expectations have partially reacted to the shadow rate over this episode. (iii) The presence of adaptive learners contributes to a de-anchoring of inflation expectations both on the upside when inflation is high, such as during the Great Inflation period, and on the downside when the ELB constraint is binding, as observed during the Great Recession period. (iv) A high share of adaptive learners and a loss of central bank credibility increase the risk of deflationary spirals. When agents start to extrapolate recent data more, the risk of deflationary spirals increases further.

At the time of writing this paper, many advanced economies have been experiencing rising and persistent inflationary pressures. This has brought many central banks' focus back to inflation expectations, with worries over potential de-anchoring risks.[2] While the

[2]See, e.g., Blanchard (2022) and the recent speech by Carstens (2022).

main focus of this paper is on business cycle dynamics at the ELB regime, the estimation results over the high-inflation period of the '60s and '70s also shed light on today's issues. The paper is organized as follows: Section 2 introduces the key model features, as well as assumptions on heterogeneous expectations and ELB, together with some theoretical results to illustrate the model properties. Section 3 discusses the estimation methodology, key results, and the empirical properties of the model. Section 4 discusses a number of counterfactual exercises at the ELB to study the interaction between heterogeneous expectations and the risk of deflationary spirals. Section 5 concludes.

1.1 Literature Review

Our paper relates to several strands of literature on adaptive learning, heterogeneous expectation, and regime switching in DSGE models. Earlier work on heterogeneous expectations in New Keynesian models considers a variety of topics; e.g., Branch (2004) studies the empirical properties of heterogeneous expectations with survey data on inflation expectations; Branch and McGough (2009) analyze the microfoundations of New Keynesian models with heterogeneous expectations; Anufriev et al. (2013) consider different interest rate rules and macroeconomic stability under heterogeneous expectations; Di Bartolomeo, Di Pietro, and Giannini (2016) study how heterogeneous expectations affect the design of optimal monetary policy in a New Keynesian model; Cornea-Madeira, Hommes, and Massaro (2019) estimate the New Keynesian Phillips curve with heterogeneous expectations; and Hommes, Massaro, and Weber (2019) test a number of heterogeneous and bounded rationality models in a learning-to-forecast experiment.

More recently, there have been a number of papers that study the interactions between the ELB, unconventional monetary policy, and heterogeneous expectations. A closely related study is Busetti et al. (2017), where the authors study how prolonged periods of weak inflation in the euro zone may induce a de-anchoring of expectations. Other closely related papers include Andrade et al. (2019), who consider forward guidance in a heterogeneous expectations framework with optimistic and pessimistic agents; Hommes and Lustenhouwer (2019), who study the theoretical properties of a New Keynesian

(NK) model with an ELB under heterogeneous expectations; Goy, Hommes, and Mavromatis (2020), who analyze the effects of different types of forward guidance in a New Keynesian model with heterogeneous expectations and the ELB constraint; Lansing (2021), where a representative agent contemplates between a targeted equilibrium and a deflationary equilibrium, and where a non-trivial probability on the deflationary equilibrium becomes partially self-fulfilling by lowering the averages of observed variables; Arifovic et al. (2020), who study heterogeneous expectations through a novel mechanism called social learning; and Carvalho et al. (2021), who estimate a model where agents are allowed to switch between decreasing and constant gain algorithms to form their expectations. The marginal contribution of our paper to this literature is to estimate the model under heterogeneous expectations together with survey data and endogenous regime switching in monetary policy (MP).

When it comes to regime-switching models, the RE framework plays a central role in the DSGE literature. These models focus on the theoretical properties and solution methods within the RE framework.[3] More recently, a number of papers also study DSGE models with endogenous regime switching under RE.[4] While there is ample research in regime-switching models with rational agents, research in this class of models with imperfect information/ learning agents has been scarce. Examples include Branch, Davig, and McGough (2007), who establish theoretical properties of learning about both regime switches and structural relations, and Gust,

[3]Examples include Farmer, Waggoner, and Zha (2009, 2011) and Cho (2016), who study the theoretical properties and determinacy conditions associated with RE equilibria in Markov-switching models; Bianchi (2016), who proposes new methods for measuring expectations and uncertainty in Markov-switching models; and Kulish and Pagan (2017), who propose solution and estimation methods for forward-looking models with structural changes under a variety of assumptions for agents' beliefs about those structural changes. Other empirical applications in regime-switching DSGE models include, among others, Sims and Zha (2006), Liu and Mumtaz (2011), Bianchi (2016), and Bianchi and Ilut (2017).

[4]See, e.g., Barthélemy and Marx (2017), who use perturbation methods to solve and estimate endogenous regime-switching models; Chang, Maih, and Fei (2018), who propose an efficient filtering method to handle the estimation of state space models with endogenous switching parameters depending on latent autoregressive factors; and Benigno et al. (2020), who consider an endogenous regime-switching framework to study financial crises.

Herbst, and Lopez-Salido (2018), who study the effectiveness of forward guidance in a model where agents are aware of regime switches but do not know the transition probabilities and instead infer about them using a form of Bayesian learning.

The paper also relates to models studying the effects of ELB and unconventional monetary policy under imperfect information and adaptive learning. Examples include Evans, Guse, and Honkapohja (2008), who study the global dynamics of liquidity traps under adaptive learning; Haberis, Harrison, and Waldron (2014), who analyze macroeconomic effects of transient interest rate pegs in an imperfect information model; Eusepi and Preston (2010), who consider central bank communication in a model where agents' expectations are not consistent with the central bank policy; Cole (2018), who studies the effectiveness of learning on forward guidance, where forward guidance is introduced into monetary policy with a sequence of shocks; and similarly Cole and Martínez-García (2019), who study the effectiveness of forward guidance in a New Keynesian model with imperfect central bank credibility. The present paper contributes to this literature by allowing a fraction of agents to use adaptive learning rules through an evolutionary selection mechanism, and by estimating the model using survey data on inflation expectations.

2. Model Setup

2.1 Structural Equations and Rational Expectations

We consider the simple canonical version of the three-equation New Keynesian model as in Clarida, Gali, and Gertler (1999).[5] We first present the skeleton form of the model without any regime switching, given by the following structural equations:

$$\begin{cases} y_t = (1-\iota_y)E_t y_{t+1} + \iota_y y_{t-1} - \frac{1}{\tau}(r_t - E_t\pi_{t+1}) + u_{y,t}, \\ \pi_t = \beta((1-\iota_p)E_t\pi_{t+1} + \iota_p\pi_{t-1}) + \kappa y_t + u_{\pi,t}, \\ r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi\pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}) + \varepsilon_{r,t}, \end{cases} \tag{1}$$

[5] Similar setups have been considered in closely related papers of Busetti et al. (2017), Goy, Hommes, and Mavromatis (2020), and Lansing (2021), among others.

where y_t, π_t, and r_t denote the output gap, inflation, and nominal interest rate, respectively. The first equation represents the IS curve, where ι_y is the intrinsic level of inertia (or indexation) in output gap, and τ is the intertemporal elasticity of substitution for households. The second equation is the Phillips curve, with ι_p the price indexation and κ denoting the slope of the Phillips curve. The last equation is the monetary policy reaction function, with ρ_r the interest smoothing rate, ϕ_π inflation reaction, ϕ_y output gap reaction, and $\phi_{\Delta y}$ output gap growth reaction. The model is supplemented with three shocks, where the demand shock $u_{y,t}$ and cost-push shock $u_{\pi,t}$ follow AR(1) processes given by

$$\begin{cases} u_{y,t} = \rho_y u_{y,t-1} + \varepsilon_{y,t}, \\ u_{\pi,t} = \rho_\pi u_{\pi,t-1} + \varepsilon_{\pi,t}. \end{cases} \tag{2}$$

The monetary policy shock $\varepsilon_{r,t}$ is assumed to be an i.i.d. process. Before introducing the ELB constraint on the nominal rates and the regime-switching setup, it is useful to start with the rational expectations (RE) equilibrium of the model, associated with the minimum state variable (MSV) solution. The model can be written in the standard matrix form:

$$\begin{cases} \boldsymbol{A} X_t = \boldsymbol{B} X_{t-1} + \boldsymbol{C} E_t X_{t+1} + \boldsymbol{D} u_t, \\ u_t = \boldsymbol{\rho} u_{t-1} + \varepsilon_t, \end{cases} \tag{3}$$

for conformable matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, and $\boldsymbol{\rho}$, with $X_t = [y_t, \pi_t, r_t]'$, $u_t = [u_{y,t}, u_{\pi,t}, 0]'$, and $\varepsilon_t = [\varepsilon_{y,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}]'$. The standard deviations of the i.i.d. shocks are denoted by $\eta_t = [\eta_y, \eta_\pi, \eta_r]'$. Under RE, the equilibrium solution takes the following form, along with the implied one-step-ahead expectations:

$$\begin{cases} X_t = \boldsymbol{b} X_{t-1} + \boldsymbol{d} u_t, \\ E_t X_{t+1} = \boldsymbol{b} X_t + \boldsymbol{d\rho} u_t. \end{cases} \tag{4}$$

Plugging the expectations back into the law of motion (3) yields

$$(\boldsymbol{A} - \boldsymbol{Cb}) X_t = \boldsymbol{B} X_{t-1} + (\boldsymbol{Cd\rho} + \boldsymbol{D}) u_t. \tag{5}$$

The RE solution is then pinned down by the following fixed-point conditions:[6]

$$\begin{cases} \boldsymbol{b} = (\boldsymbol{A} - \boldsymbol{C}\boldsymbol{b})^{-1}\boldsymbol{B}, \\ \boldsymbol{d} = (\boldsymbol{A} - \boldsymbol{C}\boldsymbol{b})^{-1}(\boldsymbol{C}\boldsymbol{d}\boldsymbol{\rho} + \boldsymbol{D}). \end{cases} \tag{6}$$

2.2 ELB and Regime Switching

In this paper our main objective is to evaluate the effects of the ELB constraint on macroeconomic outcomes. Introducing the constraint on the interest rate rule leads to the following form:

$$r_t = max\{\bar{r}, \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}) + \varepsilon_{r,t}\}, \tag{7}$$

which is an occasionally binding constraint (OBC) on the nominal rates, with $\bar{r}$ corresponding to the ELB value. In the literature, a popular method for approximating this OBC-induced non-linearity is to consider a regime-switching approach, used in, e.g., Binning and Maih (2016), Chen (2017), and Lindé, Maih, and Wouters (2017). In this setup, monetary policy is subject to two different regimes: a Taylor-rule regime where interest rates follow the intended reaction function when the ELB constraint does not bind, and an ELB regime where monetary policy becomes inactive when the reaction function becomes constrained by the lower bound. If we denote by s_t the regime-switching process, which can take on values $s_t = E$ (ELB regime) and $s_t = T$ (Taylor-rule regime), the monetary policy rule evolves according to

$$\begin{cases} r_t(s_t = T) = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \\ \qquad\qquad\qquad \phi_{\Delta y}(y_t - y_{t-1}) + \varepsilon^T_{r,t}, \\ r_t(s_t = E) = \bar{r} + \varepsilon^E_{r,t}. \end{cases} \tag{8}$$

The transition matrix is given by time-varying probabilities as follows:[7]

[6]We make use of the methods introduced in Uhlig (1995) to solve for the fixed-point conditions.

[7]For standard deviations of monetary policy shocks, we use the notation $\eta_{r,T}$ and $\eta_{r,E}$ at Taylor and ELB regimes, respectively.

$$\boldsymbol{Q}_t = \begin{bmatrix} q_t^T & 1 - q_t^T \\ 1 - q_t^E & q_t^E \end{bmatrix},$$

where the probabilities q_t^T and q_t^E depend on the central bank's desired policy rate at every period, which is defined as the shadow rate henceforth. More formally, we assume that the shadow rate r_t^* follows:

$$\begin{cases} r_t^*(s_t = T) = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}), \\ r_t^*(s_t = E) = \rho_r r_{t-1}^* + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}). \end{cases} \tag{9}$$

This structure makes use of the following assumptions: The shadow rate r_t^* is the central bank's desired level of nominal interest rate *in the absence of monetary policy shocks and the ELB constraint.* During normal times with the Taylor rule, the shadow rate is smoothed over the observed nominal interest rate. Therefore during normal times, the only difference between these two rates is the presence of i.i.d. monetary policy shocks. During ELB periods when nominal rates are constrained, the shadow rate is smoothed over itself, which allows for persistent deviations from the nominal rate beyond the i.i.d. monetary policy shocks. This captures the idea of keeping the interest rates *lower for longer*, where the central bank wants to keep the policy rate at near-ELB levels until the shadow rate recovers back to a level above the ELB.

Given the shadow rate r_t^*, the transition probabilities are determined according to

$$q_t^T = \frac{\theta_1}{\theta_1 + exp(-\Phi_1(r_t^* + (\bar{r_T} - \bar{r_E})))},$$
$$q_t^E = \frac{\theta_2}{\theta_2 + exp(\Phi_2(r_t^* + (\bar{r_T} - \bar{r_E})))}, \tag{10}$$

where $\bar{r_T}$ and $\bar{r_E}$ are the constant trend values of the nominal interest rate during Taylor and ELB regimes, respectively.[8] These parameters are introduced into the measurement equations as constants and are estimated jointly with the structural parameters of the model, which is discussed further in Section 3.

[8]Given the trend values $\bar{r_T}$ and $\bar{r_E}$, the identity for ELB constraint in (7) is given by $\bar{r} = -\bar{r_T} + \bar{r_E}$.

In a regime-switching world, the RE solution makes use of two key assumptions: Agents are aware of the current underlying regime s_t, and they know the transition matrix $\boldsymbol{Q_t}$ associated with the regimes. In other words, RE models equate agents' subjective expectations about regime switches to the objective model expectations, leading to regime-dependent expectations in the following form:

$$
\begin{aligned}
&E_t[X_{t+1}|s_t = T] = q_t^T(\boldsymbol{b}(s_{t+1} = T)X_t + \boldsymbol{d}(s_{t+1} = T)\boldsymbol{\rho} u_t) + \\
&(1 - q_t^T)(\boldsymbol{b}(s_{t+1} = E)X_t + \boldsymbol{d}(s_{t+1} = E)\boldsymbol{\rho} u_t), \\
&E_t[X_{t+1}|s_t = E] = q_t^E(\boldsymbol{b}(s_{t+1} = E)X_t + \boldsymbol{d}(s_{t+1} = E)\boldsymbol{\rho} u_t) + \\
&(1 - q_t^E)(\boldsymbol{b}(s_{t+1} = T)X_t + \boldsymbol{d}(s_{t+1} = T)\boldsymbol{\rho} u_t).
\end{aligned}
\tag{11}
$$

The RE solution in the baseline version of the model in (1) is unique and determinate when the Taylor principle of $\phi_\pi > 1$ is satisfied. The equilibrium becomes indeterminate at the ELB when monetary policy is not active. Davig and Leeper (2007) establish that in a regime-switching environment with RE, the equilibrium determinacy can continue to hold even if one of the underlying regimes is indeterminate. They define this property as the *long-run Taylor principle* (LRTP). The implications of this for the canonical New Keynesian model with Taylor and ELB regimes is that, as long as the passive (indeterminate) periods are sufficiently short lived relative to the active (determinate) periods, the model dynamics can still be characterized by a determinate equilibrium. This property allows for the estimation and simulation of RE models with regime switching in the presence of indeterminate regimes.

2.3 Heterogeneous Expectations

In this paper we deviate from the standard full rationality assumption by breaking the tight link between subjective expectations and objective model-implied expectations. In particular, we relax the assumption that agents are aware of the underlying regime s_t and the transition probability matrix $\boldsymbol{Q_t}$. We further relax the assumption that agents are rational; instead we introduce a heterogeneous expectation mechanism with anchored and de-anchored expectation rules, explained in further detail below.

2.3.1 Anchored Rational Expectations

The first type of agents form their expectations using the rational solution in (6) associated with an active Taylor rule $\phi_\pi > 1$. In other words, this type of agent always follows expectations based on a determinate RE solution. During normal times with the Taylor regime ($s_t = T$), this assumption boils down to the standard model solution associated with RE. During ELB periods ($s_t = E$), expectations associated with this type take on a different interpretation: Nominal rates are constrained by the ELB, but expectations evolve *as if* the central bank's desired interest rate path, i.e., the shadow rate r_t^*, is what matters for the economy.

The assumption that agents always use the RE solution associated with active policy rule implicitly means that they know the shadow rate at any given period, even though the shadow rate is not directly observable during ELB periods. Therefore this assumption can be interpreted as a successful central bank communication and correctly anchored expectations on the desired interest rate, which proxies for the impact of a central bank's unconventional policy tools on expectations. We assume that forward guidance communications and quantitative easing measures allow the central bank to correctly signal the desired interest rate and anchor this class of agents' expectations on the targeted equilibrium. Put differently, these agents believe that unconventional monetary policy measures perfectly substitute for the slack on the nominal rates introduced by the ELB constraint.

It is important to note that this expectation formation rule ignores not only the presence of the ELB constraint but also the presence of other agents in the economy that form their expectations differently. Therefore, these expectations correspond to a form of pseudo-rationality only, i.e., what would happen if all expectations were rational and if the monetary policy was not constrained by the ELB. Such behavior is usually referred to as a *fundamentalist* rule in heterogeneous expectations studies.[9] In this paper, we refer to this type as rational agents with *anchored* expectations.

[9] See, e.g., Hommes and Lustenhouwer (2019) and Goy, Hommes, and Mavromatis (2020), where *fundamentalist* agents use the steady-state values or long-run averages of the relevant endogenous variables when forming their expectations.

2.3.2 Adaptive Learning

The second class of agents use a constant gain recursive least squares (RLS) learning rule based on the observable variables of output gap, inflation, and nominal interest rates. Specifically, we assume that agents have the following regression model, along with the implied one-step-ahead expectations:

$$\begin{cases} X_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\beta}_{t-1} X_{t-1} + \delta_t, \\ E_t X_{t+1}^L = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\beta}_{t-1} X_t, \end{cases} \tag{12}$$

where $\boldsymbol{\alpha}_{t-1}$ is a vector of perceived means, $\boldsymbol{\beta}_{t-1}$ is the perceived first-order correlation matrix, and δ_t is a vector of i.i.d. shocks. The first equation in (12) is referred to as the agents' *perceived law of motion* (PLM) henceforth. This particular VAR(1) form of learning has been frequently used in the learning literature; see, e.g., Jääskelä and McKibbin (2010), Milani (2011), and Chung and Xiao (2013). It has the advantage of being close to the beliefs consistent with the MSV solution of the model.[10]

We use a *t-timing* assumption on expectations, which means that agents are able to use period-t information when forming their expectations. This corresponds to a joint determination of expectations and period-t variables.[11] Agents update the perceived parameters in their PLM after the endogenous variables are determined, hence these parameters appear with a lag in (12) in the form of $\boldsymbol{\alpha}_{t-1}$ and $\boldsymbol{\beta}_{t-1}$. Under constant gain RLS, the parameters evolve according to

$$\begin{cases} R_t = R_{t-1} + \gamma(\tilde{X}_{t-1}\tilde{X}_{t-1}' - R_{t-1}), \\ \Phi_t = \Phi_{t-1} + \gamma R_t^{-1} \tilde{X}_{t-1}(X_t - \Phi_{t-1}\tilde{X}_{t-1})', \end{cases} \tag{13}$$

[10] The only difference between the MSV solution and VAR(1) expectations is that in the latter, the exogenous AR(1) cost-push and demand shocks are not included in the regression. This keeps the state space of the PLM small and more tractable.

[11] The alternative is to use the assumption of $t-1$ *dating* for both types of agents, which takes on a sequential structure where first expectations are formed using information from period $t-1$ and then period-t variables are determined given the expectations. We abstract away from this approach in this paper.

where $\tilde{X}_{t-1} = [1, X'_{t-1}]'$, $\Phi_t = [\boldsymbol{\alpha}_t, \boldsymbol{\beta}_t]$, and R_t is the second moments matrix of perceived autocovariances. γ denotes the constant gain value, which determines the weight that agents place on the latest available observations. When nominal rates are constrained by the ELB, the learning rule in (13) loses its stability. During ELB regimes, we interpret the share of these agents as a measure of central bank credibility: More agents that use the anchored rational expectations rule with shadow rate reflect more trust in the central bank's ability to circumvent the ELB constraint with unconventional monetary policy tools. A lower share weakens the transmission channel from shadow rate to inflation and output gap, thereby reflecting a lower central bank credibility. A sufficiently high share of adaptive learners at the ELB creates the risk of deflationary spirals, which is illustrated in further detail in Section 2.4.

2.3.3 Aggregate Dynamics

Given the RE-based (anchored) and learning-based (de-anchored) expectation formation rules, the fraction of agents using each rule evolves according to a fitness measure based on their one-step-ahead forecasting performance as in Busetti et al. (2017), Hommes and Lustenhouwer (2019), Goy, Hommes, and Mavromatis (2020), and Lansing (2021). In particular, we assume the following fitness measures ζ_t^{RE} and ζ_t^L associated with each rule:[12]

$$\begin{cases} \zeta_t^{RE} = -(1-\omega) FE_t^{RE} + \omega \zeta_{t-1}^{RE}, \\ \zeta_t^{L} = -(1-\omega) FE_t^{L} + \omega \zeta_{t-1}^{L}, \end{cases} \tag{14}$$

where FE_t^{RE} and FE_t^L denote the sum of squared forecast errors for inflation and output gap under for the RE- and learning-based PLMs, respectively. Given the fitness measures, agents' fractions are determined by

$$n_t^{RE} = \frac{exp(\chi \zeta_t^{RE})}{exp(\chi \zeta_t^{RE}) + exp(\chi \zeta_t^{L})}, \quad n_t^{L} = \frac{exp(\chi \zeta_t^{L})}{exp(\chi \zeta_t^{RE}) + exp(\chi \zeta_t^{L})}, \tag{15}$$

[12]The fitness measures follow the standard assumption in the heterogeneous expectations literature as in the aforementioned studies.

where n_t^{RE} (rational) and n_t^L (learning) denote the fractions of agents associated with each type. χ is an *intensity of choice* measure, common across both types, which determines the frequency of switching between the rules. Finally, the implied one-step-ahead and N-step-ahead inflation expectations are given by[13]

$$\begin{cases} E_t X_{t+1} = n_{t-1}^{RE} E_t X_{t+1}^{RE} + n_{t-1}^{L} E_t X_{t+1}^{L}, \\ E_t X_{t+N} = n_{t-1}^{RE} E_t X_{t+N}^{RE} + n_{t-1}^{L} E_t X_{t+N}^{L}. \end{cases} \tag{16}$$

The model dynamics evolve according to the aggregate law of motion in (3); rational and adaptive expectations in (4) and (12); monetary policy and shadow rate rules in (8) and (9); the learning rule in (13); the rule for updating agent fractions in (14)–(15); and finally the rule to determine aggregate expectations in (16).

2.4 Adaptive Learning and Instability at the ELB: Illustration

A well-known result in the adaptive learning literature is that, akin to the determinacy condition in RE models, the learning dynamics are expectationally stable (E-stable) when the Taylor principle $\phi_\pi > 1$ is satisfied (Bullard and Mitra 2002). During ELB regimes where monetary policy is constrained, the E-stability principle breaks down for standard model parameterizations, and learning dynamics become unstable.[14]

In our heterogeneous expectation setup, the presence of adaptive learners serves as a source of potential instability at the ELB. If the share of adaptive learners becomes sufficiently high, aggregate dynamics of the model become unstable. In such an environment, adverse shocks can push the economy into self-fulfilling deflationary spirals with ever-falling inflation and output gap.

To understand the intuition behind the instability, we illustrate the key mechanism at the ELB regime in a simplified setting in order

[13]Shares of agents n_t^{RE} and n_t^L enter into aggregate expectations with a one-period lag to obtain a sequential timing structure of expectations in the model. This is discussed in further detail in Appendix D.

[14]E-stability refers to the stability of constant gain learning algorithms. When the E-stability condition is not satisfied, learning dynamics are characterized by divergent behavior; see Evans and Honkapohja (2001) for further details.

to obtain analytical stability conditions. Consider the three-equation model in (1) without shocks and lagged state variables:

$$\boldsymbol{A}X_t = \boldsymbol{C}E_t X_{t+1}, \tag{17}$$

with $\boldsymbol{A} = \begin{bmatrix} 1+\frac{\phi_y}{\tau} & \frac{\phi}{\tau} \\ -\kappa & 1 \end{bmatrix}$ and $\boldsymbol{C} = \begin{bmatrix} 1 & \frac{1}{\tau} \\ 0 & \beta \end{bmatrix}$. Under RE, the agents' PLM takes the form of $E_t X_{t+1} = \boldsymbol{a}$. Plugging back into the law of motion and solving for the equilibium yields $\boldsymbol{a} = \boldsymbol{0}$ as the unique RE solution if the Taylor principle $\phi_\pi > 1$ is satisfied. Under adaptive learning, agents' PLM is time varying:

$$E_t X_{t+1} = \boldsymbol{\alpha}_{t-1}, \tag{18}$$

where the vector $\boldsymbol{\alpha}_{t-1}$ is updated every period as new observations become available. Assuming shares of adaptive learners $\boldsymbol{n_{t-1}^L} = [n_{\pi,t-1}^L, n_{y,t-1}^L]'$ for inflation and output gap, respectively, the implied actual law of motion (ALM) is given by[15]

$$\boldsymbol{A}X_t = \boldsymbol{C}[\boldsymbol{n_{t-1}^L}\boldsymbol{\alpha}_{t-1} + (\boldsymbol{1} - \boldsymbol{n_{t-1}^L})\boldsymbol{a}]. \tag{19}$$

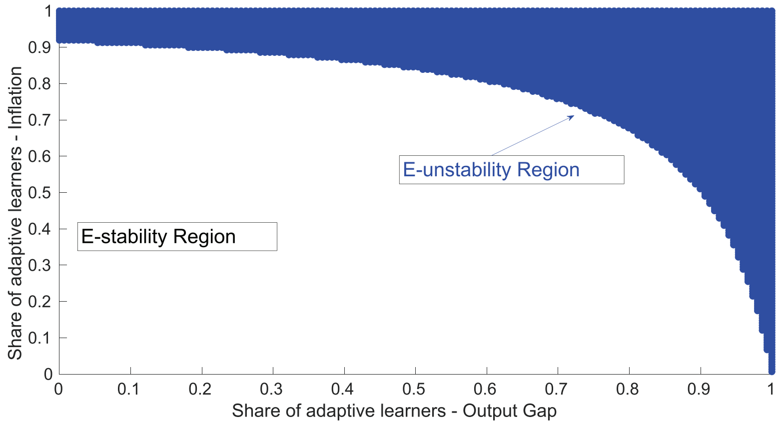
The T-map associated with the adaptive agents' PLM is given by $\boldsymbol{\alpha}_{t-1} \Rightarrow T(\boldsymbol{\alpha}_{t-1}) = \boldsymbol{\Gamma_1 n_t^L}\boldsymbol{\alpha}_{t-1}$, with $\boldsymbol{\Gamma_1} = \boldsymbol{A}^{-1}\boldsymbol{C}$.[16] The law of motion is E-stable under learning if all eigenvalues of $\frac{\partial T(\boldsymbol{\alpha}_{t-1})}{\partial \boldsymbol{\alpha}_{t-1}} = \boldsymbol{\Gamma_1 n_{t-1}^L}$ have real parts less than 1.

In this simple setting, when the state variables X_t deviate from the deterministic equilibrium $X_t = \boldsymbol{0}$, the forecasting performance of adaptive learners outpaces those of rational agents. This is due to the adaptive learners adjusting their beliefs based on their previous errors, while rational agents' forecasts remain fixed at the equilibrium. Consequently, the share of adaptive learners increases and the central bank loses credibility whenever the state variables move away from the equilibrium. In turn, the lower credibility and the high

[15] In our empirical section, we use the restriction $n_{\pi,t}^L = n_{y,t}^L$. In this section, for illustrative purposes, we allow for different shares of adaptive learning agents for inflation and output gap $[n_{\pi,t}^L, n_{y,t}^L]$.

[16] T-map refers to the mapping from agents' PLM to the implied ALM of the model. See Evans and Honkapohja (2001) for a detailed treatment.

Figure 1. E-unstability Region in the Skeleton New Keynesian Model as a Function of the Share of Adaptive Learners



Note: The blue area shows the region where learning dynamics become unstable in the model.

share of adaptive learners slow down the economy's return to equilibrium.[17] As we will show in Section 3, the same principle applies to the empirical estimates of inflation expectations under RE and learning. While expectations under RE tend to be centered around the equilibrium, those of adaptive learners follow the data more closely and become de-anchored during periods when inflation persistently deviates from its trend.

The instability in the model arises when persistent deviations from the equilibrium coincide with the ELB constraint on nominal interest rates. In these scenarios, the rising share of adaptive learners is combined with the central bank's inability to combat the falling inflation and output gap. As a result, the economy is stuck in a self-fulfilling deflationary spiral at the ELB, which generates a de-anchoring of inflation expectations and a loss of central bank credibility.

Figure 1 shows the instability region in the model as a function of the share of adaptive learners on inflation and output

[17] In Appendix A, we derive the analytical relationship between agent shares and forecast errors for the simple law of motion (19).

gap.[18] As the proportion of adaptive learners increases significantly for either inflation or output gap expectations, the system becomes E-unstable. In Section 4, we delve into a more detailed discussion of the potential occurrence of these scenarios using our full-fledged model estimated for the United States.

3. Estimation

3.1 Methodology and Data

This section discusses the estimation methodology, along with the data set used in estimations and prior distributions for estimated parameters. The regime-switching model described in the previous section can be summarized as a state space system with time-varying matrices as follows:

$$S_t = \boldsymbol{\gamma}^{s_t}_{1,\Phi_{t-1}} + \boldsymbol{\gamma}^{s_t}_{2,\Phi_{t-1}} S_{t-1} + \boldsymbol{\gamma}^{s_t}_{3,\Phi_{t-1}} \varepsilon_t, \tag{20}$$

with $S_t = [X_t, \varepsilon_t]'$ and conformable matrices $\boldsymbol{\gamma}^{s_t}_{1,\Phi_{t-1}}$, $\boldsymbol{\gamma}^{s_t}_{2,\Phi_{t-1}}$, and $\boldsymbol{\gamma}^{s_t}_{3,\Phi_{t-1}}$ with two layers of time variation in the system matrices. The time-varying adaptive learning parameters $\boldsymbol{\alpha}_{t-1}$, $\boldsymbol{\beta}_{t-1}$ and shares of agents n^L_{t-1}, n^{RE}_{t-1} are captured by Φ_{t-1}. Monetary policy regime switches (ELB regime or Taylor rule) are captured by s_t. The timing assumptions of the expectations in the model admit a conditionally linear structure, where the likelihood is evaluated using the Kim and Nelson (1999, henceforth KN) filter. The parameters are estimated using standard Bayesian methods; see Appendix D for further details of the implementation.[19]

To estimate the model, we use historical U.S. data on output gap, inflation, and nominal interest rates over the period 1960:Q1–2019:Q4.[20] The output gap series is obtained by detrending GDP

[18] We use a standard parameterization with $\beta = 0.99$, $\tau = 1$, and $\kappa = 0.05$ for this illustration. The instability boundary in Figure 1 depends on the parameter values, but the main intuition is robust to alternative parameterizations.

[19] The working paper version of this study (Özden 2021) provides an alternative estimation method, where the heterogeneity of expectations is approximated as an endogenous regime-switching mechanism and the model is rewritten in a four-regime setup.

[20] We also use three years of data over 1957:Q1–1959:Q4 as a burn-in sample to initialize the likelihood.

using the method proposed in Hamilton (2018).[21] We further use short-term (one-quarter) and long-term (10-year) inflation expectations as observables in our estimation.

There is a wide variety of survey data on inflation expectations, with their availability ranging over different sample periods. In this paper, we utilize the ATSIX index introduced in Aruoba (2020). This is a composite index combining data from the Survey of Professional Forecasters (SPF) (Croushore 1993) and Blue Chip forecasts to obtain a reliable term structure of inflation expectations. The index has the advantage of avoiding the fixed-horizon versus fixed-event issues that are prevalent in many surveys, and also yields better forecasts of realized inflation than its alternatives as outlined in Aruoba (2020).[22] ATSIX series are available from 1992:Q1 for long-run expectations, and from 1998:Q1 for short-run expectations. For the earlier sample over 1960:Q1–1991:Q4, we treat inflation expectations as latent variables when estimating the model in order to test the model's predictions about these series during the Great Inflation period. To discuss model-implied expectation dynamics, we splice the ATSIX index with data from the SPF, which allows us to extend the expectation series back to 1979:Q1. We use the combined series to qualitatively examine model-implied inflation expectations over the early part of our sample. We use the following measurement equations in the estimation:

$$\begin{cases} y_t = y_t^{obs}, \\ \pi_t = \bar{\pi} + \pi_t^{obs}, \\ r_t = \bar{r}(s_t) + r_t^{obs}, \\ E_t\pi_{t+1} = \bar{\pi} + E_t\pi_{t+1}^{obs,ATSIX}, \\ E_t\pi_{t+40} = \bar{\pi} + E_t\pi_{t+40}^{obs,ATSIX}, \end{cases} \tag{21}$$

[21] Appendix F provides a sensitivity check around alternative measures of output gap, where we reestimate the model with a quadratically detrended output gap as in Cornea-Madeira, Hommes, and Massaro (2019), and output gap based on the Congressional Budget Office's (CBO's) measure of potential output (Shackleton 2018).

[22] In many surveys the forecasters are not consistently asked about their forecasts over a fixed horizon but rather over a fixed event, which can lead to an inconsistency about the timing assumptions. The ATSIX index does not suffer from this drawback; see Aruoba (2020) for further details.

where the right-hand-side variables are the historical data (observables), and the left-hand-side variables are the model variables. To include inflation expectations data in the estimation, we introduce two measurement error shocks. The law of motion for one-quarter and 10-year inflation expectations becomes[23]

$$\begin{cases} E_t\pi_{t+1} = n_{t-1}^{RE} E_t\pi_{t+1}^{RE} + n_{t-1}^{L} E_t\pi_{t+1}^{L} + \varepsilon_{\pi,t}^{exp,1}, \\ E_t\pi_{t+40} = n_{t-1}^{RE} E_t\pi_{t+40}^{RE} + n_{t-1}^{L} E_t\pi_{t+40}^{L} + \varepsilon_{\pi,t}^{exp,40}. \end{cases} \tag{22}$$

We assume a constant trend inflation $\bar{\pi}$ and a regime-switching constant trend interest rate $\bar{r}(s_t)$, which takes on values $\bar{r}_T$ and $\bar{r}_E$ as shown in (10). This approach closely follows that of Gust, Herbst, and Lopez-Salido (2018), who assume a shift in the intercept of interest rate $\bar{r}(s_t)$, which switches to a lower value during the ELB period.[24] We further impose the inflation trend $\bar{\pi}$ on measurement equations for inflation expectations. The constants are included in the measurement equations and are estimated along with the structural parameters, rather than detrending the data prior to estimation.

We estimate three additional models together with the heterogeneous expectation model: (i) the RE benchmark, without adaptive learners and with no regime switching in monetary policy; (ii) the RE model with regime switching in monetary policy; and (iii) a pure adaptive learning model with regime switching in monetary policy. Together, these models help us disentangle the marginal impact of adaptive learning, heterogeneous expectations, and monetary policy switching on model fitness.[25]

[23]Standard deviations of the measurement errors on inflation expectations are denoted by $\eta_{\pi,exp}^{SR}$ (short term) and $\eta_{\pi,exp}^{LR}$ (long term), respectively.

[24]The same intercept shift is also assumed for the shadow rate over the same period.

[25]The regime-switching RE model is approximated with a constant transition matrix $\boldsymbol{Q}$ when we estimate the model, to avoid the non-linearity induced by the expectational equations in (11). The heterogeneous expectation and adaptive learning models are instead estimated with the time-varying matrix $\boldsymbol{Q_t}$. Since expectations do not directly interact with the transition matrix in these models, their estimation still admits a conditionally linear structure that can be handled by the standard Kim and Nelson (1999) filter. Özden and Wouters (2021) show that the impact of a time-varying transition matrix has a negligible impact on

All structural, learning, and switching parameters are assigned prior distributions consistent with previous values used in the literature. This is discussed in detail in Appendix C, and Table C.1 provides a summary of all distributions used. The initial beliefs for heterogeneous expectations and adaptive learning models are derived from the estimated RE model, where we first estimate the baseline model in (1) under RE without regime switching. Using the estimated RE model, we retrieve the implied VAR(1) beliefs consistent with the estimated equilibrium, which are used to initialize the beliefs of adaptive learners. We use Sims's (1999) *csminwel* algorithm to obtain the posterior mode, which is used to initialize the MCMC algorithm with random-walk Metropolis-Hastings. We use 500,000 parameter draws for all models under consideration. The first 50 percent of the draws are discarded as a burn-in sample, and highest posterior density (HPD) intervals are computed using the remaining 50 percent of the sample.

3.2 Posterior Estimation Results

In this section we discuss the posterior estimation results for the heterogeneous expectation (HE) model along with the three accompanying models described in the previous section, i.e., (i) baseline rational expectations (RE), (ii) rational expectations with regime switching (RE-RS), and (iii) adaptive learning (AL).

The posterior moments of parameter distributions, together with the marginal likelihoods of all models, are reported in Table 1.[26] Based on the marginal likelihoods and Bayes factors, three key results emerge: First, all three models with regime switching in monetary policy fit the data better than the RE benchmark, regardless of the underlying expectation mechanism (i.e., rational, learning, or heterogeneous expectations). Second, both the HE and the AL model perform better than the RE-RS model. This suggests that the presence of adaptive learners improves the model fitness. Third, the HE model performs better than the AL model, which shows

estimation results, therefore the results with constant matrix $\boldsymbol{Q}$ and time-varying matrix $\boldsymbol{Q_t}$ are comparable.

[26]The (log-) marginal likelihood values reported in the table are based on the modified harmonic mean estimator. The Bayes factors are calculated using a log base 10, following Jeffreys Guidelines (Greenberg 2012).

Table 1. Posterior Distribution Moments for Baseline RE (No Switching), RE with Switching in MP, Heterogeneous Expectations, and Adaptive Learning Models

	RE			RE—Switching in MP			Hetero Expectations			Learning		
Parameter	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**
$\overline{\pi}$ (Inflation Trend)	0.63	0.66	0.69	0.62	0.64	0.65	0.55	0.58	0.6	0.03	0.15	0.27
$\bar{r_T}$ (Int. Rate Trend—Taylor)	0.67	1.02	1.43	0.78	1	1.24	0.25	0.26	0.27	0.32	0.33	0.34
$\bar{r_E}$ (Int. Rate Trend—ELB)				0.01	0.01	0.01	0.03	0.03	0.04	0.029	0.032	0.035
κ (NKPC Slope)	0.0018	0.0033	0.0071	0.0011	0.0018	0.0028	0.0017	0.0032	0.0052	0.009	0.014	0.017
τ (Risk Aversion)	0.43	0.68	1.16	1.55	2.19	3.05	1.03	1.13	1.3	1.22	1.28	1.43
ι_y (Indexation—IS Curve)	0.15	0.22	0.3	0.09	0.15	0.21	0.03	0.06	0.12	0.63	0.69	0.74
ι_p (Indexation—NKPC)	0.03	0.07	0.15	0.02	0.05	0.08	0.09	0.15	0.2	0.36	0.44	0.52
ϕ_π (MP Inflation Reaction)	1.18	1.45	1.77	1.16	1.41	1.72	1.24	1.53	1.79	1.08	1.33	1.55
ϕ_y (MP Output Gap Reaction)	0.19	0.28	0.39	0.17	0.26	0.38	0.34	0.42	0.51	0.23	0.31	0.44
$\phi_{\Delta y}$ (MP Output Gap Growth Reaction)	0.11	0.15	0.19	0.06	0.09	0.11	0.09	0.12	0.14	0.03	0.05	0.06
ρ_r (MP Smoothing)	0.87	0.9	0.92	0.87	0.9	0.93	0.9	0.92	0.94	0.91	0.93	0.95
ρ_y (Persistence—Demand Shock)	0.96	0.97	0.98	0.88	0.92	0.95	0.93	0.95	0.96	0.22	0.24	0.28
ρ_π (Persistence—Supply Shock)	0.05	0.2	0.43	0.01	0.04	0.08	0.01	0.04	0.08	0.02	0.05	0.12
η_y (St. Dev.—Demand Shock)	0.2	0.31	0.48	0.13	0.17	0.22	0.28	0.32	0.36	0.91	1.01	1.11
η_π (St. Dev.—Supply Shock)	0.3	0.44	0.58	0.65	0.71	0.77	0.35	0.38	0.41	0.34	0.37	0.41
$\eta_{r,T}$ (St. Dev.—MP Shock Taylor)	0.22	0.26	0.31	0.2	0.22	0.24	0.22	0.24	0.27	0.19	0.2	0.22
$\eta_{r,E}$ (St. Dev.—MP Shock at ELB)				0.03	0.03	0.04	0.01	0.01	0.01	0.008	0.01	0.013
$\eta^{SR}_{\pi,exp}$ (St. Dev.—SR Inflation Exp.)	0.09	0.16	0.24	0.09	0.1	0.11	0.09	0.11	0.12	0.13	0.15	0.17
$\eta^{LR}_{\pi,exp}$ (St. Dev.—LR Inflation Exp.)	0.08	0.09	0.10	0.05	0.06	0.07	0.07	0.08	0.09	0.11	0.12	0.13
Φ_1 (MP Switching—Taylor to ELB)							0.06	0.1	0.15	0.11	0.2	0.39
Φ_2 (MP Switching—ELB to Taylor)							0.07	0.16	0.31	0.09	0.15	0.26
γ (Constant Gain)							0.0584	0.0585	0.0587	0.0575	0.0579	0.0581
ω (Memory)							0.49	0.62	0.74			
χ (Intensity of Choice)							0.38	0.51	0.66			
$1 - q^T$ (Exog. Exit Probability—Taylor)			0.13	0.19	0.27							
$1 - q^E$ (Exog. Exit Probability—ELB)			0.03	0.04	0.06							
Marg. (log-) Likl.		–479.63			–435.86			–410.06			–432.66	
Bayes Factor		1			19			29.86			20.39	

Note: The estimation period is from 1960:Q1 to 2019:Q4 using historical U.S. data.

that the expectational heterogeneity mechanism also improves the model fit. Taken together, these results suggest that both monetary policy switching and heterogeneity of expectations are important mechanisms to fit the data.[27]

Before analyzing the model-implied dynamics and inflation expectations, we discuss the differences in estimated parameter values. First, comparing the baseline and regime-switching RE models, it is readily seen that most parameters have similar posterior HPD intervals. There are two exceptions: First, the estimated slope of the Phillips curve κ is lower in the regime-switching model, implying a higher degree of price stickiness when the ELB constraint is accounted for. This is in line with the findings in Del Negro, Giannoni, and Schorfheide (2015), Lindé, Smets, and Wouters (2016), and Lindé, Maih, and Wouters (2017). Second, the risk-aversion parameter τ is considerably higher in the regime-switching model than in the baseline. This higher value is explained by the expectational feedback channel in the IS curve: When monetary policy is constrained by ELB, agents' expectations take into account the constraint in the regime-switching model. Therefore the ex ante real interest rate $r_t - E_t[\pi_{t+1}]$ has a larger feedback effect on output gap y_t in the IS equation once the ELB constrained is accounted for. The higher risk-aversion parameter in the regime-switching model has the effect of dampening this feedback channel.

Next we compare the HE and AL models with the regime-switching RE. The differences are more pronounced in this comparison: NKPC is steeper in the HE model, and it becomes even more steep in the AL model. The estimated NKPC slope is in line with previous findings in adaptive learning literature; e.g., Milani (2007), Jääskelä and McKibbon (2010), and Slobodyan and Wouters (2012b) all report lower Calvo parameters or steeper NKPC slope in their estimation results under learning compared with the RE benchmark. This result suggests that learning dynamics can partially substitute for nominal price stickiness. The risk-aversion parameter τ is lower in both the HE and the AL model compared with the RE-RS model, which relates to the expectational feedback channel discussed above:

[27] As a robustness check, in Appendix G we provide estimations of all models without inflation expectations. The relative ranking of the models remains the same.

Agents in the RE-RS model switch their expectations immediately once the economy becomes constrained by the ELB, which strengthens the feedback channel from ex ante real interest rate to output gap. In the HE and AL models, expectations adapt gradually over time as agents learn about the consequences of the ELB. Therefore the resulting risk aversion τ is lower than the RE-RS model but still higher than the baseline RE model.

The constant trend parameters for inflation and interest rate in measurement equations, $\bar{\pi}$ and $\bar{r_T}$, are lower in HE and AL models compared with RE and RE-MS. This is due to the time variation about the perceived mean $\boldsymbol{\alpha}_t$ in the HE and AL models.[28] While agents' expectations about the mean of inflation and output gap are zero in the RE and RE-RS models, the time-varying intercepts in the HE and AL models introduce a non-zero mean in their expectations. In other words, the HE and AL models have an endogenous inflation and interest rate trend induced by time-varying beliefs. This results in a level shift and lower estimates for the intercepts in measurement equations. The results are in line with, e.g., Carvalho et al. (2021), who interpret time-varying learning dynamics as a source of endogenous inflation trend.

The remaining structural parameter estimates are similar under all models, with HPD bands well within the range of each other. The posterior means for ϕ_π range over the interval [1.33, 1.53], whereas the output gap reaction ϕ_y and output gap growth reaction $\phi_{\Delta y}$ range over the intervals [0.26, 0.42] and [0.05, 0.15], respectively. The same argument also applies to interest rate smoothing ρ_r, which fluctuates between 0.90 and 0.95. All models except AL are characterized by a highly persistent demand shock (ρ_y ranging between [0.92, 0.96]) and a near-white-noise supply shock (ρ_π ranging between [0.04, 0.2]). This is accompanied by low indexation parameters in these models, with ι_y ranging over $[0.06, 0.22]$ and ι_π over $[0.05, 0.15]$. The AL model is instead characterized by a less persistent demand shock with $\rho_y = 0.24$, which is substituted by higher indexation parameters $\iota_y = 0.69$ and $\iota_\pi = 0.44$. Some studies in the past have suggested that learning dynamics in DSGE models

[28] The time variation in all PLM coefficients, $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}_t$, is reported in Figure E.1, Appendix E.

can substitute for mechanical sources of persistence such as indexation, habits, capital adjustment costs, and persistence of structural shocks. Other studies have found learning dynamics have a negligible impact on these parameter estimates.[29] Hence the evidence in the literature on the impact of learning dynamics on mechanical sources of persistence is mixed and depends on the particular model setup. In our setting, learning and heterogeneity do not substitute for mechanical sources of persistence.

In the HE and AL models, the estimated constant gain values have similar posterior means with 0.0585 and 0.579, respectively. This implies that approximately 50 percent of adaptive learners' expectations are determined by three years of most recent data.[30] For the HE model the estimated memory parameter ω in expectational switching is 0.6, whereas the intensity of choice χ is 0.51. Our estimated constant gain value is somewhat higher than other studies in the literature that have only used aggregate macrodata in their estimation. Furthermore, our intensity of choice χ is significantly lower than other studies that have estimated similar mechanisms in the absence of inflation expectations, e.g., Cornea-Madeira, Hommes, and Massaro (2019). Therefore our findings suggest that using inflation expectations in the estimation is crucial for correctly identifying the parameters that determine the expectation formation process.[31]

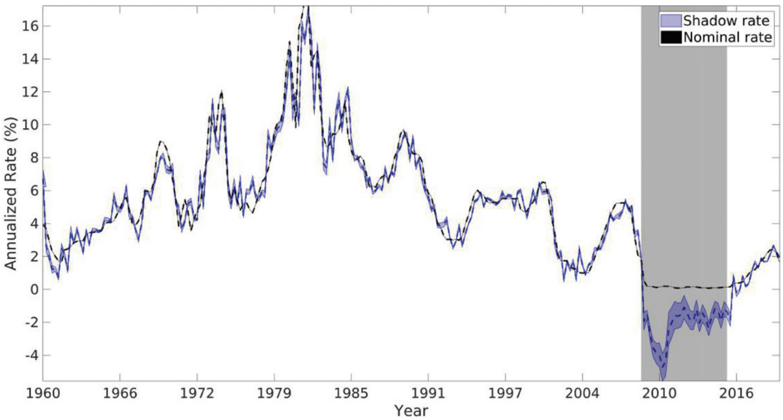
For the remainder of this section, we discuss model-implied dynamics of the HE model to better understand whether and how it generates more realistic expectation formation dynamics. Figure 2 shows the 90 percent HPD band of the estimated shadow rate under

[29]For example, Milani (2007) documents that learning dynamics result in substantially lower degrees of habit and indexation. Examples of papers that do not find important differences in estimated RE and learning models include Jääskelä and McKibbon (2010) and Slobodyan and Wouters (2012a).

[30]This suggests a geometric discount rate of $(1-\gamma)^T$ for T periods in the past.

[31]As a robustness check, in Appendix G we report estimation results without using any inflation expectations data. This yields a lower gain coefficient and a significantly higher intensity of choice with a larger uncertainty band. This provides further support for the argument that having inflation expectations data as observables plays an important role in identification of learning- and heterogeneity-related parameters.

Figure 2. Estimated Shadow Rate Together with the Nominal Interest Rates for United States over the Period 1960:Q1–2019:Q4



Note: The blue area depicts the 90 percent HPD band of the shadow rate estimate. The gray area depicts the ELB regime following the GFC.

the HE model, together with nominal interest rates over our estimation sample.[32] The estimated shadow rate is crucial in determining the expectations of rational agents in the model. It is readily seen that during the Taylor-rule regime before the Great Recession, the shadow rate closely follows the nominal interest rate path. As discussed in the previous section, this close relationship is by construction since the shadow rate is smoothed over the observed nominal rate during Taylor regime. The rates start diverging when the economy enters the ELB regime, and the shadow rate reaches a trough in 2010:Q2 with a range of [3.9, 5.56]. This is consistent with other studies in the literature, e.g., Kulish, Morley, and Robinson (2014), where the authors report an annual rate of –4 percent as the trough of their shadow rate estimate. The rate starts to gradually pick up after the initial crisis period, and the two rates converge again by the end of 2015 as nominal rates starts rising and the economy

[32]To obtain the HPD bands for the shadow rate and other latent variables in the model, we simulate the model 1,000 times using parameter values from the MCMC chain.

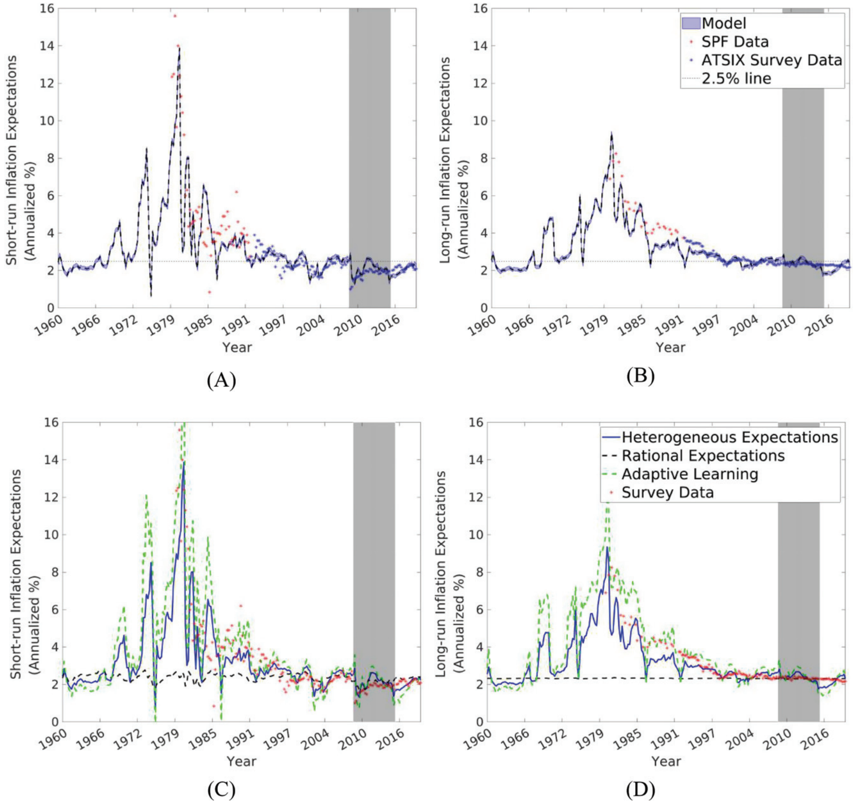
switches back to the Taylor-rule regime. The observed pattern in the shadow rate is also consistent with other empirical studies, e.g., Aruoba et al. (2022), who use a structural VAR with occasionally binding constraints to estimate the shadow rate.

Figure 3 shows survey expectations data together with model-implied short- and long-run inflation expectations.[33] The top two panels show the results with the heterogeneous expectations model: The blue areas depict the 90 percent HPD interval of short-run (panel A) and long-run (panel B) inflation expectations for the HE model. The figures include two types of survey data: ATSIX index (blue line) is used in the estimation of the model; it is available from 1992:Q1 for long-run (10-year) and 1998:Q1 for short-run (one-quarter) inflation expectations. Data from the SPF (red line) are not included in the estimation. We use this to splice the ATSIX index and examine the model-implied inflation expectations over the earlier part of the sample. It is readily seen that model-implied inflation expectations match the survey data fairly well. In particular, over the Great Inflation period, the model captures the de-anchoring of inflation expectations very well. Over the period where SPF data is available (i.e., 1979:Q1 onwards), model-implied series match the survey data closely for both short- and long-run inflation expectations, despite the fact that no inflation expectations data are included in the estimation over this period.

To see how well the HE model performs in terms of model-implied expectations, panels C–D in Figure 3 compare the median model-implied short- and long-run inflation expectations under the RE, AL, and HE models against survey data. This comparison helps us understand whether the improvement in model fitness for the HE model is accompanied by a better fit on inflation expectations. Two results become evident from these figures: First, the RE model falls short of explaining the survey data during the Great Inflation period, when inflation was high and inflation expectations were de-anchored. In particular, long-run inflation expectations under RE remain stable throughout the entire sample, regardless of the realized inflation. The AL and HE models are both more successful along

[33] By model-implied expectations, we refer to expectation series generated by the models in absence of measurement errors.

Figure 3. Short-Term and Long-Term Inflation Expectations over the Period 1960:Q1–2019:Q4



Note: Panels A and B show the 90 percent HPD bands of model-implied expectations (short- and long-run expectations, respectively) from the HE model, together with the ATSIX index, SPF data, and a constant 2.5 percent line. Panels C and D compare the posterior medians of model-implied expectations under the RE, HE, and AL models against survey data (ATSIX combined with SPF).

this dimension, and they both match periods with de-anchored inflation expectations fairly well. Second, the AL model typically has more trouble during periods with relatively stable inflation over the post–Great Moderation period. Model-implied data from the '80s onwards are typically too volatile, particularly for long-run inflation expectations. The HE model overcomes these two shortcomings by

Table 2. In-Sample RMSEs and Biases for Inflation Expectations in RE, HE, and AL Models

	Hetero. Exp.	Rational Exp.	AL
Long-Run Inflation Expectations			
RMSE	0.59	1.4	0.73
Bias	–0.21	–0.72	0.22
Short-Run Inflation Expectations			
RMSE	1.43	2.54	1.68
Bias	–0.19	–0.94	0.31

Note: The sample period for expectations covers 1979:Q1–2019:Q4, which consists of SPF data between 1979:Q1 and 1991:Q4, and ATSIX index between 1992:Q2 and 2019:Q4.

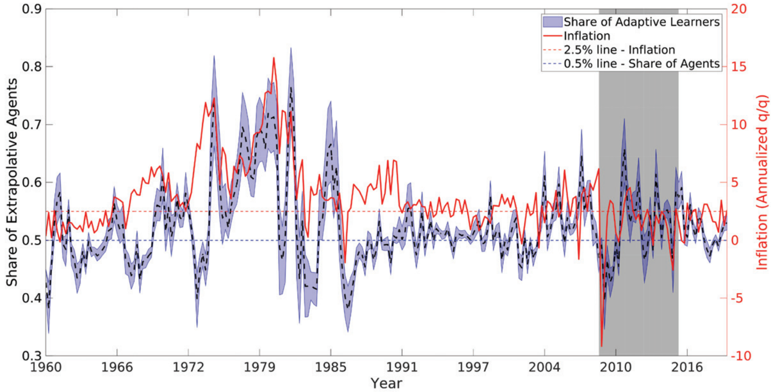
allowing the agents to endogenously switch between learning and rational expectations.

To make this point more clear, in Table 2 we report the in-sample root mean square errors (RMSEs) and biases of inflation expectation forecast errors for the RE, HE, and AL models. Not surprisingly, the RE model yields the worst statistics both for short-run and for long-run inflation expectations. On average, RE-implied expectations are negatively biased due to the models' inability to produce de-anchored expectations over the Great Inflation period. The AL model yields better RMSEs and biases than the RE benchmark, but it is still outperformed by the HE model. In particular, the AL model suffers from positive biases on average, suggesting that it tends to overshoot the degree of de-anchoring. These results confirm that having both types of expectations with endogenous shares is vital for explaining periods of both de-anchored and anchored inflation expectations.

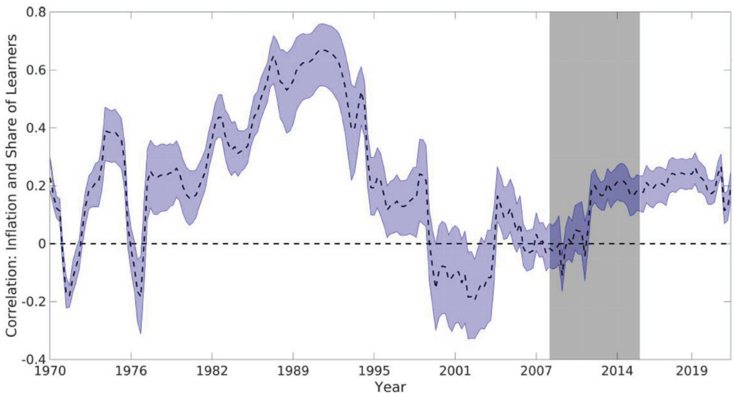
Figure 4 shows the estimated share of adaptive learners over history together with inflation (panel A), and their correlation over the sample period (panel B).[34] To understand how the estimate of these shares are pinned down, recall from Section 2.4 that there is

[34] To calculate the correlation series, we use a rolling sample of 10 years, starting with the sample over 1960:Q1–1969:Q4.

Figure 4. Estimated Share of Adaptive Learners over History



(A)



(B)

Note: Panel A shows the estimated 90 percent HPD band of the share of adaptive learners (left y-axis) and CPI inflation (right y-axis) over the estimation period 1960:Q1–2019:Q4. Panel B shows the estimated 90 percent HPD band of rolling-window correlation between the share of adaptive learners and CPI inflation. The correlations are based on a sample size of 10-year rolling window.

a tight relationship between agents' forecast errors and their shares. Inflation expectations under the RE benchmark tend to gravitate toward the inflation target, whereas those of adaptive learners follow realized inflation more closely, as is readily seen in Figure 3.

Consequently, periods where inflation is persistently above the trend, such as during the Great Inflation period, are typically dominated by a high share of adaptive learners. By incorporating data on both inflation and inflation expectations in the estimation, the model is able to identify the parameters governing the speed of learning γ, the intensity of choice χ, and the degree of memory in switching ω. These parameters collectively determine persistence and volatility of the estimated share of adaptive learners based on agents' realized forecast errors.

During the initial part of the sample, the co-movement between the share of adaptive learners and inflation is remarkably high, with a correlation of up to 0.78 during the '80s. The model explains the high inflation and de-anchored inflation expectations over this period with a high share of adaptive learners. As adaptive learners extrapolate recent data to form their expectations, this has a tendency to put further upward pressure on inflation. When inflation starts to stabilize after the 1990s, the tight positive correlation between inflation and share of adaptive learners breaks down. During the early 2000s, we observe a reversal in the correlation, which becomes weakly negative until 2008. A higher share of adaptive learners creates a weak deflationary effect over this period with stable inflation, before changing signs again following the GFC. At the beginning of GFC the share of adaptive learners temporarily falls down, which is partly how the model explains the missing deflation puzzle. During this period adaptive learners expect a stronger deflation that is not observed in the data, and the model explains this as a temporary fall in their share. Throughout the rest of the Great Recession the shares remain balanced around 50 percent, which suggests that expectations have at least partially remained anchored and responded to the shadow rate over this period. This is in line with other empirical studies in the literature, e.g., Mavroeidis (2021), who suggest that inflation and output gap have partially responded to shadow rate over the post-GFC period. This result can be interpreted as a successful central bank communication by the Federal Reserve Board over this period.

From a narrative perspective, model dynamics under heterogeneous expectations suggest that endogenous central bank credibility plays an important role in driving inflation. During the Great Inflation period, the model shows that the share of adaptive learners is high and central bank credibility is low. As the central bank brings inflation under control, the share of adaptive learners stabilizes around 50 percent from the '90s onwards. These are are in line with previous studies on the subject. For example, Carvalho et al. (2021) analyze a model where agents switch between a constant gain and a decreasing gain learning rule. They find that constant gain learning was dominant during the early '70s and '80s, whereas decreasing gain learning has become more prevalent from the '90s onwards, which is consistent with our results.

Along similar lines, Malmendier and Nagel (2016) provide a demographic interpretation in a model where agents overweight inflation experienced during their lifetimes. In this context, the authors document a divergence in expectations between younger and older cohorts during the late '70s and '80s. Younger individuals' experience with high inflation over this period contributes to a high perceived inflation persistence, which in turn creates more persistence and sluggishness in inflation expectations. This demographic dispersion in inflation expectation only goes away in the '90s. In our framework, this is reflected as a declining share of adaptive learners. The authors show that learning from experience can be seen as a microfoundation of constant gain learning models, since aggregate dynamics from the model can be approximated quite closely with a constant gain mechanism. As such, the success of our heterogeneous expectation model in explaining survey data, as well as the dominance of constant gain learners over the Great Inflation period, can be interpreted as a validation of their learning-from-experience framework in a DSGE setup. Lower average inflation over the Great Recession period, combined with the ELB constraint, creates a risk of de-anchored inflation expectations in the negative direction. We study this channel in further detail in the next section.

4. Model Dynamics at the ELB

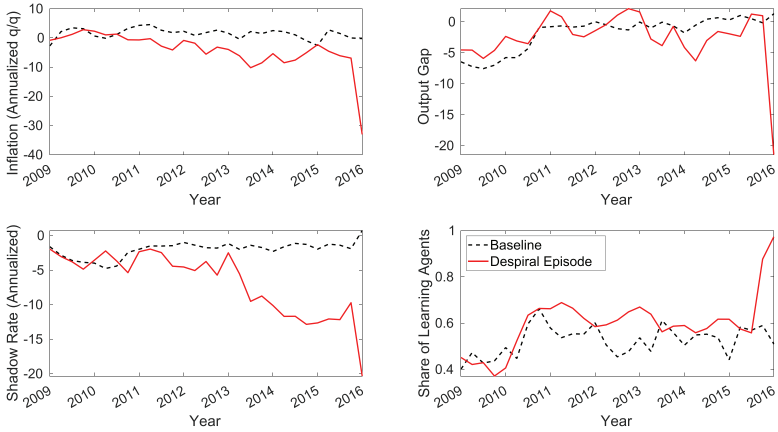
The estimation results in the previous section highlight that the HE model fits the data better, and the heterogeneity mechanism is crucial in explaining the historical inflation dynamics. As shown in Section 2.4, the expectational switching mechanism creates the possibility of observing deflationary spirals at the ELB when the share of adaptive learners becomes too high. With this in mind, in this section we focus on the ELB regime over the post-GFC period and investigate the properties of ELB and deflationary spiral episodes in the heterogeneous expectation model. The discussion is focused on two key questions: (i) What is the risk of a deflationary spiral occurring in the model? (ii) How do ELB regimes and deflationary spirals interact with the heterogeneity mechanism and endogenous central bank credibility?

We use two exercises to analyze these issues. In the first exercise, we use U.S. data in 2008:Q4 as a starting point and generate density forecasts between 2009:Q1 and 2016:Q4 at the estimated posterior mean values. This is helpful to understand the estimated risk of de-anchoring and deflationary spirals, and how this risk interacts with key parameters that determine the learning and switching mechanism. In the second exercise, we use standard stochastic simulations of the model to discuss key moments and statistics at ELB episodes. This is useful to discuss the model-implied unconditional distributions of ELB and deflationary spiral episodes. In both exercises, we formally define a deflationary spiral as an episode where quarter-on-quarter inflation falls below 10 percent.[35]

Starting with the density forecasts of the model, Figure 5 shows an example of a deflationary spiral in the heterogeneous expectation model. Following the GFC period, the share of adaptive learners remains above the estimated baseline for an extended period from 2010:Q1 onwards and deflationary pressures keep building up. The shadow rate becomes increasingly more accommodative. Due to falling inflation and the ELB constraint on nominal rates, real interest rates rise and depress aggregate demand. As agents lose their trust in the central bank's ability to make up for this increasingly large slack in nominal interest rates, more agents switch to the adaptive learning rule. When the share of adaptive learners becomes critically high, the economy enters into a deflationary spiral episode with ever-falling inflation and output gap. This is an illustration

[35]We use 1,000 simulations in both exercises to compute the HPD bands. The shocks are drawn from normal distributions using the estimated standard deviations at the posterior mean in Table 1. For stochastic simulations we use a maximum simulation length of 5,000 periods, and simulations are terminated in both exercises when a deflationary spiral is detected. Further note that the parameters are fixed at the posterior mean in both exercises, therefore the confidence bands reported in this section do not reflect any parameter uncertainty.

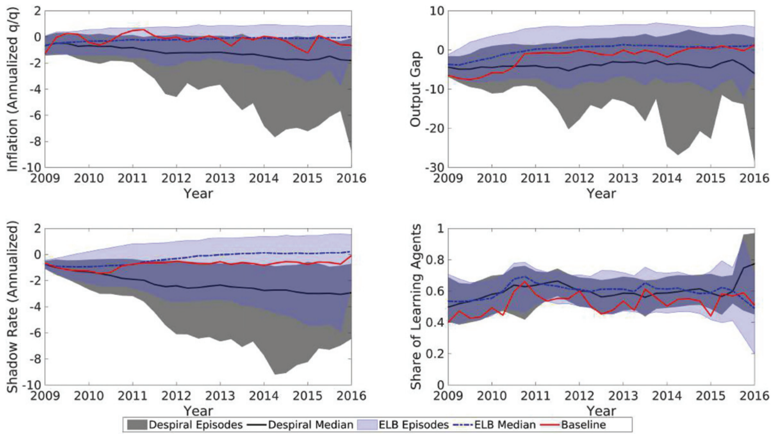
Figure 5. Example of Deflationary Spiral Occurring over the Period 2009:Q1–2016:Q4



of the analytical results discussed in Section 2.4. While the stability conditions are no longer tractable in the full model, the density forecasts and stochastic simulations of the model show that the main intuition continues to hold in a more empirically relevant setup.

To see how often and under what conditions these deflationary spirals occur in the model, Figure 6 shows the 90 percent HPD interval of density forecasts from 2009:Q1 onwards. We divide the simulations into two categories when reporting the confidence bands: episodes that result in a deflationary spiral (gray area), and all other ELB episodes that do not result in deflationary spirals (blue area). It is readily seen that despiral episodes are characterized by a large downside risk on not only inflation but also output gap and shadow rate. More importantly, despiral episodes are characterized by a higher average share of adaptive learners, i.e., lower central bank credibility. It is also worth noting that the median forecasts under non-spiral ELB episodes are close to realized inflation, output gap, and the estimated values of shadow rate and share of adaptive learners. Baseline results for all variables fall within the range of 90 percent HPD interval over the forecast horizon. This suggests that unconventional monetary policy actions over this period have kept the share of adaptive learners low enough to make a switch to a deflationary spiral episode unlikely.

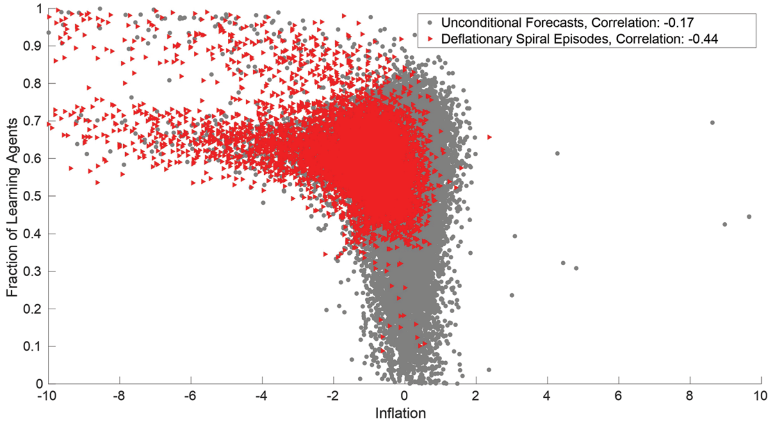
Figure 6. Density Forecasts with Heterogeneous Expectations Model over the Period 2009:Q1–2016:Q4



Note: The 90 percent HPD bands are reported for simulations that result in a deflationary spiral (gray area) and standard ELB episodes that do not result in deflationary spirals (blue area). The purple area corresponds to the region where the HPD bands overlap.

To make the connection between the share of adaptive learners and deflationary outcomes more concrete, Figure 7 shows the distribution of inflation against the share of adaptive learners over the counterfactual period. The unconditional distributions (i.e., both despiral and non-spiral episodes), depicted by gray dots, are characterized by a weakly negative correlation between inflation and share of adaptive learners (−0.17). In these simulations, inflation does not fall strongly into the negative territory and therefore an increase in the share of adaptive learners only creates a weak deflationary effect. On the contrary, deflationary spiral episodes, depicted by red dots, are associated with not only a higher share of adaptive learners and lower central bank credibility but also a stronger negative correlation between inflation and the share of adaptive learners (−0.44). When credibility is low to begin with (i.e., share of learners is high), a further decline in credibility tends to create a stronger deflationary effect than a starting point with high credibility and low share of adaptive learners.

Figure 7. Distribution of Inflation (x-axis) Plotted Against the Distribution of the Share of Adaptive Learners (y-axis)



Note: We report the distributions for all ELB episodes (gray area) and deflationary spirals (red area) separately.

Table 3. Probability of Despiral Episodes under Alternative Parameterizations of the Model

Scenario	**Deflationary Spiral Probability**
Baseline	27.6%
Large Gain ($\gamma = 0.1$)	38.8%
High Intensity of Choice ($\chi = 2$)	46.6%

Table 3 shows the estimated probability of a despiral episode underlying the density forecasts in Figures 6 and 7. At the estimated parameter values, 27.6 percent of the simulations result in deflationary spirals. This is accompanied by two additional parameterizations of the model: If we increase the constant gain parameter γ to 0.1 from its estimate of 0.0585, i.e., when adaptive learners pay more attention to recent data, then the probability of deflationary spirals increases to 38.8 percent. If we change the intensity of choice parameter χ to 2 from its estimate of 0.51, i.e., agents switch more frequently between rational expectations and adaptive learning, then the probability increases further to 46.6 percent. Both counterfactuals represent scenarios where expectations can become de-anchored more easily, and as a result they both result in more frequent deflationary spirals.

Our second exercise is based on unconditional stochastic simulations of the model as discussed above. This helps us examine model properties and key statistics associated with ELB regimes, and it serves as a robustness check to see if the results based on U.S. data continue to hold in an unconditional environment. The simulations are mainly characterized by short-lived ELB episodes, with occasional long-lived ELB episodes: The probability that an ELB episode lasts for at least one, two, and five years are 27.5 percent, 12 percent, and 1.5 percent, respectively. The corresponding distributions of ELB episodes, together with other related summary statistics such as the frequency of ELB episodes and how long it takes to encounter deflationary spirals, are discussed in further detail in Appendix E. Here we instead focus on the averages of key variables at the ELB and despiral episodes, reported in Table 4: the top two columns in the table show the averages across all ELB and despiral episodes. Similar to density forecasts, despiral episodes are on average characterized by lower inflation, output gap, shadow rate, and both short-run and long-run expectations, together with a substantially higher average share of adaptive learners (0.8) compared with non-spiral ELB regimes (0.54). The bottom two columns in the table show the average *entry* values into ELB and despiral episodes. When the economy enters into an ELB regime with an already high share of adaptive learners, i.e., low central bank credibility, then the regime is more likely to turn into a deflationary spiral. Taken together, these results confirm the takeaways from U.S.-based density forecasts in an unconditional setting.

Our results in this section show that the share of adaptive learners and initial beliefs in ELB regimes play an important role in driving deflationary spirals in the model. It is important to highlight the difference between deflationary spirals in our endogenous central bank credibility setup and those that have been studied in a fully rational setup. Most notably, Bianchi, Melosi, and Rottner (2021) study deflationary spirals in a rational expectations model,

Table 4. Averages and Average Entry Values into ELB and Despiral Regimes

	Average—ELB	**Average—Defl. Spiral**
Inflation	–0.21	–1.26
Inf. Exp.—SR	–0.15	–1.16
Inf. Exp.—LR	–0.03	–0.29
Shadow Rate	–1.56	–7.16
Output Gap	–5.04	–29.52
Output Gap Exp.	–4.46	–27.68
Fraction of Learners	0.54	0.8
	Average Entry—ELB	**Average Entry—Defl. Spiral**
Inflation	–0.12	–0.44
Inf. Exp.—SR	–0.1	–0.36
Inf. Exp.—LR	–0.02	–0.07
Shadow Rate	–1.31	–2.43
Output Gap	–3.84	–9.19
Output Gap Exp.	–3.33	–8.01
Fraction of Learners	0.54	0.63

Note: The results are based on 1,000 stochastic simulations of the model.

where agents' expectations about future ELB regimes may lead to a deflationary bias. When the possibility of hitting the ELB regime becomes too large, the deflationary bias increases. For sufficiently large values of the bias, the equilibrium loses its determinacy and deflationary spirals occur. In this fully rational environment, the central bank can mitigate the risk of deflationary spirals by implementing an asymmetric monetary policy rule, whereby its response to inflation above target is slower than its response to inflation below target. This emphasizes the channel of monetary policy rule in mitigating the risk of deflationary spirals, where an asymmetric rule reduces the risk of encountering ELB episodes. Our model instead emphasizes the central bank credibility channel: Deflationary spirals occur when agents lose their trust in the central bank's ability to circumvent the ELB constraint through unconventional monetary policy measures. Therefore the risk of deflationary spirals can be mitigated by managing expectations at the ELB through central bank communication channels.

5. Conclusions

In this paper we estimate a heterogeneous expectation model based on the canonical New Keynesian model, with monetary policy subject to the ELB constraint on nominal interest rates. We use aggregate macrodata as well as survey data on inflation expectations to identify the learning and heterogeneous switching mechanisms in the model. Several results stand out. The heterogeneous expectation model fits the data better than models with fully rational agents or with agents using only adaptive learning. The results suggest that private-sector inflation expectations in the United States over the sample period 1960:Q1–2019:Q4 can be described as a mixture of anchored, rational expectations and de-anchored expectations based on adaptive learning. The latter plays a particularly important role during high inflation periods with de-anchored expectations, such as the Great Inflation period. The model also shows that during the U.S. experience with ELB after the GFC, expectations have remained partially anchored and responded to the shadow rate. Third and most importantly, our counterfactual experiments show that a high degree of de-anchoring and a loss of central bank credibility are associated with an increased likelihood of deflationary spirals and prolonged recessions. This emphasizes the importance of central bank communication channels in managing expectations and mitigating deflationary spiral risk. The paper also opens potential avenues of future research. The current framework only incorporates unconventional monetary policy through its expectational channel. Future studies should also account for the direct effects of unconventional tools, in particular quantitative easing measures. Moreover, the heterogeneous expectation and endogenous central bank credibility framework laid out in this paper is likely to have important insights into the liftoff from the ELB, and for the post-pandemic inflationary environment that many central banks in advanced economies have been experiencing.

Appendix A. Forecast Errors and Shares of Agents

In this section we use the simple deterministic version of the three-equation model described in Section 2.4 to derive an analytical relationship between agents' forecast errors and their shares when the model deviates from equilibrium. Recall that the economy's law of motion is given by

$$\boldsymbol{A}X_t = \boldsymbol{C}[\boldsymbol{n^L_{t-1}}\boldsymbol{\alpha}_{t-1} + (1 - \boldsymbol{n^L_{t-1}})\boldsymbol{a}],$$

with $\boldsymbol{a} = \mathbf{0}$ in equilibrium. We first rewrite the adaptive learning rule given in (13), which can be simplified in the absence of stochastic shocks and lagged state variables. Given that agents are only learning about the intercepts in this case, we have $\tilde{X}_{t-1} = \boldsymbol{c}$ and $\Phi_t = \boldsymbol{\alpha}_t$, where $\boldsymbol{c}$ is a vector of constants. Without loss of generality, set $\boldsymbol{c} = 1$ and rewrite the perceived volatility term R_t as follows:

$$R_t = R_{t-1} + \gamma(1 - R_{t-1}) = \gamma\sum_{j=0}^{t-1}(1-\gamma)^j + R_0,$$

for some initial value R_0. As $t \to \infty$, we get $R_t = \gamma\sum_{j=0}^{\infty}(1-\gamma)^j = 1$. Using this, the equation for $\boldsymbol{\alpha}_t$ can be simplified as $t \to \infty$:

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \gamma R_t^{-1}(X_t - \boldsymbol{\alpha}_{t-1}) = \gamma\sum_{j=0}^{\infty}(1-\gamma)^j X_{t-j}.$$

Then it follows that $\frac{\partial\boldsymbol{\alpha}_t}{\partial X_{t-j}} = \gamma(1-\gamma)^j > 0$ for any constant gain value $\gamma > 0$. In other words, whenever X_t deviates from the equilibrium, agents revise their beliefs about $\boldsymbol{\alpha}_t$ in the same direction as X_t.

Given the forecasting rules, we rewrite agents' shares in terms of their forecast errors. Note that rational and adaptive agents' squared forecast error vectors are given by X_t^2 and $(X_t - \boldsymbol{\alpha}_{t-1})^2$, respectively. Setting $\chi = 1$ and $\omega = 0$ in the switching function without loss of generality, Equations (14) and (15) together reduce to

$$\boldsymbol{n_t^{RE}} = \frac{exp(-X_t^2)}{exp(-X_t^2) + exp(-(X_t - \boldsymbol{\alpha}_{t-1})^2)},$$
$$\boldsymbol{n_t^{L}} = \frac{exp(-(X_t - \boldsymbol{\alpha}_{t-1})^2)}{exp(-X_t^2) + exp(-(X_t - \boldsymbol{\alpha}_{t-1})^2)},$$

with $\boldsymbol{n_t^{RE}} = [n_{\pi,t}^{RE}, n_{y,t}^{RE}]'$ and $\boldsymbol{n_t^{L}} = [n_{\pi,t}^{L}, n_{y,t}^{L}]'$. Given the fact that $\frac{\partial \boldsymbol{\alpha}_t}{\partial X_{t-j}} > 0$, we have $exp(-X_t^2) < exp(-(X_t - \boldsymbol{\alpha}_{t-1})^2)$. This implies $\boldsymbol{n_t^{L}} > \boldsymbol{n_t^{RE}}$ whenever X_t deviates from the equilibrium ($X_t^2 > 0$). Therefore any deviations from the equilibrium are met with a rising share of adaptive learners until the economy converges back to the equilibrium.

Appendix B. Data Descriptions

This section describes the quarterly time series used in the estimations. The data set spans from 1960:Q1 to 2019:Q4 and all time series except inflation expectations are retrieved from the Federal Reserve Economic Data (FRED) database.

- Real Gross Domestic Product (FRED mnemonic: GDPC1), denoted as GDP_t and available at https://fred.stlouisfed.org/series/GDPC1.
- Consumer Price Index for All Urban Consumers (FRED mnemonic: CPIAUCSL), denoted as P_t and available at https://fred.stlouisfed.org/series/CPIAUCSL.
- Effective Federal Funds Rate (FRED mnemonic: FEDFUNDS), denoted as R_t and available at https://fred.stlouisfed.org/series/FEDFUNDS.
- CBO's Measure of Real Potential GDP (FRED mnemonic: GDPPOT), denoted as GDP_t^{pot} and available at https://fred.stlouisfed.org/series/GDPPOT.
- Aruoba Term Structure of Inflation Expectations, denoted as $ATSIX_t$ and available at https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/atsix. We use the notation $ATSIX_t^{t+j}$ to refer to the measure of j-quarter-ahead forecasts made at period t.

The following variables are used in the measurement equations:

- Output gap y_t^{obs} is based on the cycle component of the Hamilton filter, applied to $log(GDP_t)$ over the estimation sample. For the CBO-based measure of output gap, which is used as a robustness check in Appendix F, output gap is computed as $y_t^{obs} = log(GDP_t) - log(GDPPOT_t)$.
- Inflation $\pi_t^{obs} = \frac{P_t}{P_{t-1}}$.
- Nominal interest rate $r_t^{obs} = R_t$.
- Short-term (one-quarter-ahead) inflation expectations $E_t\pi_{t+1}^{obs,ATSIX} = ATSIX_t^{j+1}$.
- Long-term (10-year-ahead) inflation expectations $E_t\pi_{t+40}^{obs,ATSIX} = ATSIX_t^{j+40}$.

Appendix C. Prior Distributions

This section discusses the prior distributions of all structural parameters used in the estimation. Table C.1 provides a summary all parameter distributions.

The risk-aversion parameter τ has a gamma distribution with a mean 2 and standard deviation 0.5 as in An and Schorfheide (2007). The monetary policy reaction coefficients are all based on the Smets-Wouters (2007) model. Accordingly, inflation reaction ϕ_π is assigned a gamma distribution with mean 1.5 and standard deviation 0.25; output gap reaction coefficients ϕ_y and $\phi_{\Delta y}$ are assigned gamma distributions with mean 0.25 and standard deviation 0.1. The interest rate smoothing parameter ρ_r is assigned a beta distribution with mean 0.75 and standard deviation 0.1. Similarly, shock parameters are based on the same model, where shock persistence parameters ρ_y and ρ_π are assigned a beta distribution with mean 0.5 and standard deviation 0.2, and shock standard deviations are assigned inverted gamma distributions with mean 0.1 and standard deviation 2. The standard deviation of the monetary policy shock over the ELB regime is an exception, which is instead assigned a uniform distribution over the unit interval. For the slope of the Phillips curve κ, we use a relatively tight prior of a beta distribution with mean 0.05 and standard deviation 0.025.

Table C.1. Prior Distributions for the Estimated Parameters in the New Keynesian Model

Parameter	Dist.	Prior Mean	Prior St. Dev.	Lower B.	Upper B.
$\overline{\pi}$ (Inflation Trend)	Uniform	0.5	0.29	0	∞
$\bar{r_T}$ (Int. Rate Trend—Taylor)	Uniform	0.5	0.29	0	∞
$\bar{r_E}$ (Int. Rate Trend—ELB)	Normal	0.1	0.25	0	∞
κ (NKPC Slope)	Beta	0.05	0.025	0	1
τ (Risk Aversion)	Gamma	2	0.5	0	∞
ι_y (Indexation—IS Curve)	Beta	0.25	0.1	0	1
ι_π (Indexation—NKPC)	Beta	0.25	0.1	0	1
ϕ_π (MP Inflation Reaction)	Gamma	1.5	0.25	1	∞
ϕ_y (MP Output Gap Reaction)	Gamma	0.25	0.1	0	∞
$\phi_{\Delta y}$ (MP Output Gap Growth Reaction)	Gamma	0.25	0.1	0	∞
ρ_y (Persistence—Demand Shock)	Beta	0.5	0.2	0	1
ρ_π (Persistence—Supply Shock)	Beta	0.5	0.2	0	1
ρ_r (MP Smoothing)	Beta	0.5	0.2	0	1
η_y (St. Dev.—Demand Shock)	Inv. Gamma	0.1	2	0	∞
η_π (St. Dev.—Supply Shock)	Inv. Gamma	0.1	2	0	∞
$\eta_{r,T}$ (St. Dev.—MP Shock at Taylor)	Inv. Gamma	0.1	2	0	∞
$\eta_{r,E}$ (St. Dev.—MP Shock at ELB)	Uniform	0.5	0.29	0	∞
$\eta_{\pi,exp}^{SR}$ (St. Dev.—SR Inflation Exp.)	Inv. Gamma	0.1	2	0	∞
$\eta_{\pi,exp}^{LR}$ (St. Dev.—LR Inflation Exp.)	Inv. Gamma	0.1	2	0	∞
Φ_1 (MP Switching—Taylor to ELB)	Gamma	0.2	0.1	0	∞
Φ_2 (MP Switching—ELB to Taylor)	Gamma	0.2	0.1	0	∞
γ (Constant Gain)	Gamma	0.035	0.015	0	1
ω (Memory)	Beta	0.5	0.2	0	1
χ (Intensity of Choice)	Gamma	5	2	0	∞
$1-q^T$ (Exog. Exit Probability—Taylor)	Uniform	0.5	0.29	0	1
$1-q^E$ (Exog. Exit Probability—ELB)	Uniform	0.5	0.29	0	1

This corresponds to a lower mean and standard deviation compared with previous studies; e.g., An and Schorfheide (2007) use a wider beta distribution with mean 0.3 and standard deviation 0.15. Nevertheless, the prior used here encompasses parameter values consistent with most empirical studies as its credible interval. The indexation parameters ι_y and ι_π are assigned beta distributions with mean 0.25 and standard deviation 0.1. The constant trend parameters in the measurement equations are assigned uniform distributions over the interval [0.2], except for the output gap trend, which is fixed at 0 and is not included in the estimation. The constant trend for interest rates during the ELB period, $\bar{r_E}$, is assigned a more informative normal prior with a mean of 0.1 and standard deviation 0.25 in order to restrict the range of parameter values over this period. For the constant transition probabilities in the RE model, $1 - q^T$ and $1 - q^E$, we assign uniform priors over the unit interval.[36] These parameters correspond to the exit probabilities from Taylor and ELB regimes, respectively. For the endogenous switching models, the parameters θ_1 and θ_2 in the monetary policy switching functions are fixed at 1. For the other two parameters on monetary policy switching, we assign gamma distributions with mean 0.2 and standard deviation 0.1 on $\frac{\Phi_1}{1000}$ and $\frac{\Phi_2}{1000}$, which covers both gradual and abrupt transitions for monetary policy regime switching. The persistence of expectational switching, ω, is assigned the same distribution as the shock persistence parameters, i.e., a beta distribution with mean 0.5 and standard deviation 0.2. The intensity of choice χ is assigned a gamma distribution with mean 5 and standard deviation 2, which is based on the findings of Cornea-Madeira, Hommes, and Massaro (2019) on inflation expectations. Finally, the constant gain parameter γ is assigned a gamma distribution with mean 0.035 and standard deviation 0.015, which is based on Slobodyan and Wouters (2012b).

[36] This differs from previous studies that assume tighter beta distributions, e.g., Chen (2017) and Lindé, Maih, and Wouters (2017).

Appendix D. System of Equations and Timing Assumptions

The full system of equations characterizing the heterogeneous expectation model is as follows:

$$\begin{cases} \underline{\text{Law of motion:}} \\ \boldsymbol{A}(s_t)X_t = \boldsymbol{B}(s_t)X_{t-1} + \boldsymbol{C}(s_t)E_tX_{t+1} + \boldsymbol{D}(s_t)u_t, \\ u_t = \boldsymbol{\rho}u_{t-1} + \varepsilon_t, \\ \underline{\text{Expectations:}} \\ E_tX_{t+1} = n_{t-1}^{RE}E_tX_{t+1}^{RE} + n_{t-1}^{L}E_tX_{t+1}^{L}, \\ E_tX_{t+1}^{RE} = \boldsymbol{b}X_t + \boldsymbol{d\rho}u_t, \\ E_tX_{t+1}^{L} = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\beta}_{t-1}X_t, \\ \underline{\text{Agent shares, fitness and forecast errrors:}} \\ n_t^{RE} = \frac{exp(\chi\zeta_t^{RE})}{exp(\chi\zeta_t^{RE})+exp(\chi\zeta_t^{L})}, \\ n_t^{L} = \frac{exp(\chi\zeta_t^{L})}{exp(\chi\zeta_t^{RE})+exp(\chi\zeta_t^{L})}, \\ \zeta_t^{RE} = -(1-\omega)FE_t^{RE} + \omega\zeta_{t-1}^{RE}, \\ \zeta_t^{L} = -(1-\omega)FE_t^{L} + \omega\zeta_{t-1}^{L}, \\ FE_t^{RE} = (X_t - E_{t-1}X_t^{RE})^2, \\ FE_t^{L} = (X_t - E_{t-1}X_t^{L})^2, \\ \underline{\text{Adaptive Learning:}} \\ R_t = R_{t-1} + \gamma(\tilde{X}_{t-1}\tilde{X}_{t-1}' - R_{t-1}), \\ \Phi_t = \Phi_{t-1} + \gamma R_t^{-1}\tilde{X}_{t-1}(X_t - \Phi_{t-1}\tilde{X}_{t-1})'. \end{cases} \quad (D.1)$$

(D.2)

The intraperiod timing structure of the model at period t is as follows:

- Given state variables X_{t-1} and regime transition matrix $\boldsymbol{Q_{t-1}}$ from period $t-1$, the shadow rate r_t^*, new transition matrix $\boldsymbol{Q_t}$, and new regime probabilities $(s_t = T)$ and $(s_t = E)$ are realized.
- State variables X_t and expectations E_tX_{t+1} are jointly determined, given beliefs $\boldsymbol{\alpha}_{t-1}$, $\boldsymbol{\beta}_{t-1}$; share of agents n_{t-1}^{RE}, n_{t-1}^{RE} from period $t-1$; and regime probabilities $(s_t = T)$ and $(s_t = E)$.

- Given new state variables X_t, forecast errors FE_t^{RE}, FE_t^{L}; fitness measures ζ_t^{RE}, ζ_t^{L}; and new shares of agents n_t^{RE}, n_t^{L} are realized.
- Given new state variables X_t, beliefs of adaptive learners $\boldsymbol{\alpha}_t$, $\boldsymbol{\beta}_t$ are updated.

We use a modified version of the Kim-Nelson filter (KN filter) to estimate the latent variables, regime probabilities, and the likelihood function. Given the sequential timing of events in the model, the filter admits a conditionally linear structure and consists of the following main blocks: (i) a standard Kalman filter to estimate the latent variables for given beliefs and agent shares, (ii) a Hamilton filter to estimate the latent regime probabilities (Taylor regime or ELB), (iii) a collapsing step to average out the state variables and state covariance matrix, and (iv) updating agent fractions and beliefs conditional on the collapsed state variables. Then the Kalman-filter steps of the next period are applied conditional on the updated fractions and beliefs. Further details of the filtering approach can be found in the appendix of Özden and Wouters (2021).

Appendix E. Additional Model Statistics

This appendix reports additional results related to the HE model. Recall that adaptive learners' PLM is assumed to take the following VAR(1) form:

$$X_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\beta}_{t-1} X_{t-1} + \delta_t, \tag{E.1}$$

where $X_t = [y_t, \pi_t, r_t]'$, $\boldsymbol{\alpha}_{t-1} = [\alpha_\pi^{t-1}, \alpha_y^{t-1}, \alpha_r^{t-1}]'$, and

$$\boldsymbol{\beta}_{t-1} = \begin{bmatrix} \beta_{y,y}^{t-1} & \beta_{y,\pi}^{t-1} & \beta_{y,r}^{t-1} \\ \beta_{\pi,y}^{t-1} & \beta_{\pi,\pi}^{t-1} & \beta_{\pi,r}^{t-1} \\ \beta_{r,y}^{t-1} & \beta_{r,\pi}^{t-1} & \beta_{r,r}^{t-1} \end{bmatrix}.$$

Figure E.1 shows the estimated time variation in the PLM coefficients of adaptive learners throughout the sample period 1960:Q1–2019:Q4. Figure E.2 shows some additional summary statistics from the stochastic simulations of the HE model discussed in Section 4.

Figure E.1. Estimated Time Variation in PLM Coefficients of Adaptive Learners in the Heterogeneous Expectation Model over the Period 1960:Q1–2019:Q4

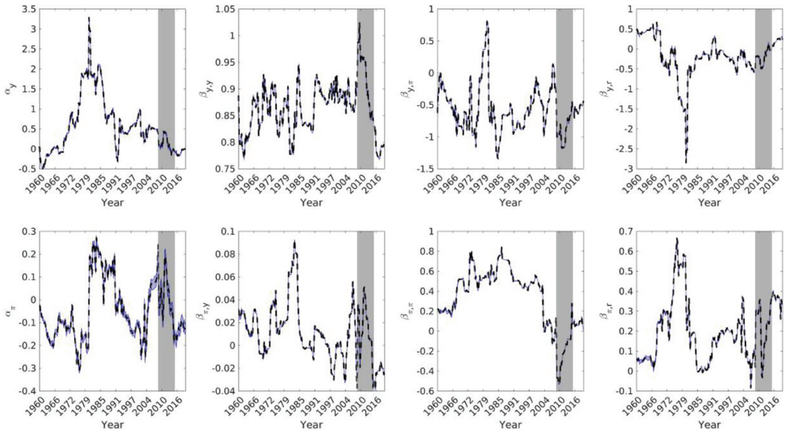
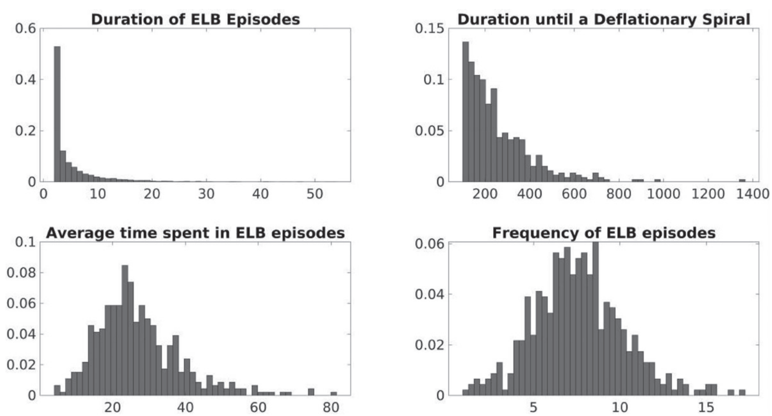


Figure E.2. Summary Statistics from Stochastic Simulations of the Model at the ELB



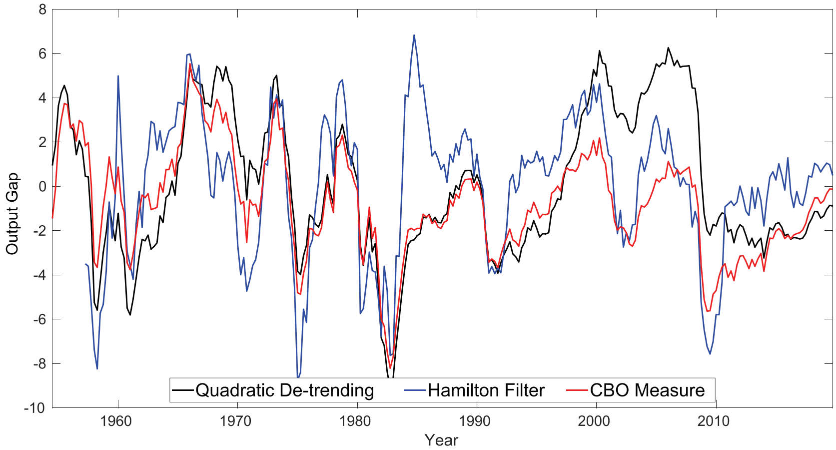
Note: The results are based on 1,000 simulations of the model.

In the simulations, the average duration of an ELB regime (top-left panel) is 3.5 quarters, with a probability of 1.5 percent for durations exceeding five years. In other words, most ELB regimes are short lived, mixed in with the occasional long-lived ELB regimes. The top-right panel shows the distribution of durations until a deflationary spiral is observed in the model. While not all ELB regimes result in deflationary spirals, as discussed in Section 4, all simulations eventually result in a deflationary spiral when the economy is hit with a large enough shock to push the share of adaptive learners into a critically high value. On average, it takes 257 quarters in the model for a deflationary spiral to occur. The bottom-left panel shows the average time spent in ELB regimes in the model: A simulation spends 25 percent of its duration in ELB regimes on average. The bottom-right panel shows the frequency of ELB regimes in the simulations: On average, the model encounters six ELB regimes once every 100 quarters. The frequency and duration of ELB regimes in the model are generally in the range of numbers reported in the literature. For example, Hills, Nakata, and Schmidt (2016) report a range of 10–27 percent as the time spent in the ELB regime in their model calibrated for the U.S. economy. Similarly, Chu and Zhang (2022) report a range of 16–29 percent of ELB regimes in Bank of Canada's main DSGE model ToTEM under a variety of monetary policy rules.

Appendix F. Robustness: Alternative Measures of Output Gap

In this appendix, as a robustness check, we discuss the estimation results of the heterogeneous expectation model under alternative measures of output gap. As discussed in Section 3, our baseline measure of output gap utilizes the Hamilton filter. This is constructed by computing the cyclical component based on the two-year-ahead forecast error of the series using a random-walk model; see Hamilton (2018) for further details. We provide two alternative measures to this output gap. The first one is based on a simple quadratic detrending of real GDP series, as in, e.g., Cornea-Madeira, Hommes, and Massaro (2019). The second one is based on the CBO's estimate of potential output, where output gap is computed as the difference between output and its potential. The resulting measures of output gap are shown in Figure F.1, whereas the parameter estimates for the model under alternative measures are reported in Table F.1. All three measures are qualitatively similar and generally agree over

Figure F.1. Alternative Measures of Output Gap Based on Hamilton Filter, CBO's Measure of Output Gap, and Quadratically Detrended Output over the Period 1960:Q1–2019:Q4



periods with excesss demand and excess supply. The measure based on the Hamilton filter is more volatile than its alternatives, suggesting that the estimated trend (i.e., potential output) under this filter is smoother. The results in Table F.1 suggest that the parameter estimates are generally robust to alternative measures of output gap. There are a few exceptions: The NKPC slope κ and risk aversion τ are both higher under the Hamilton filter, whereas indexation in IS curve ι_y is lower compared with its alternatives. All of these are consequences of the more volatile and less persistent output gap measure under the Hamilton filter. The remaining parameter estimates are very similar across different measures, with parameter bands well within the range of each other.

Table F.1. Posterior Distribution Moments for the Heterogeneous Expectation Model with Alternative Measures of Output Gap, Estimated with Data over Period 1960:Q1–2019:Q4

	Hamilton Filter			CBO Measure of Potential			Quadratic Detrending		
Parameter	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**
$\overline{\pi}$ (Inflation Trend)	0.55	0.58	0.6	0.55	0.58	0.61	0.53	0.55	0.59
$\bar{r_T}$ (Int. Rate Trend—Taylor)	0.26	0.26	0.26	0	0.13	0.3	–0.15	0	0.2
$\bar{r_E}$ (Int. Rate Trend—ELB)	0.03	0.03	0.04	0.03	0.03	0.04	0.03	0.03	0.04
κ (NKPC Slope)	0.0017	0.0032	0.0052	0.0006	0.0014	0.0028	0.0001	0.0003	0.0007
τ (Risk Aversion)	1.03	1.13	1.3	0.53	0.61	0.71	0.82	0.99	1.12
ι_y (Indexation—IS Curve)	0.03	0.06	0.12	0.4	0.45	0.51	0.27	0.36	0.46
ι_p (Indexation—NKPC)	0.09	0.15	0.2	0.05	0.09	0.13	0.07	0.12	0.17
ϕ_π (MP Inflation Reaction)	1.24	1.53	1.79	1.34	1.71	2.15	1.4	1.65	1.83
ϕ_y (MP Output Gap Reaction)	0.34	0.42	0.51	0.23	0.34	0.52	0.22	0.3	0.39
$\phi_{\Delta y}$ (MP Output Gap Growth Reaction)	0.09	0.12	0.14	0.22	0.25	0.28	0.2	0.26	0.3
ρ_y (Persistence—Demand Shock)	0.9	0.92	0.94	0.93	0.95	0.97	0.93	0.94	0.96
ρ_π (Persistence—Supply Shock)	0.93	0.95	0.96	0.92	0.95	0.97	0.94	0.96	0.98
ρ_r (MP Smoothing)	0.01	0.04	0.08	0.01	0.02	0.06	0.01	0.05	0.1
η_y (St. Dev.—Demand Shock)	0.28	0.32	0.36	0.38	0.42	0.5	0.26	0.32	0.39
η_π (St. Dev.—Supply Shock)	0.35	0.38	0.41	0.34	0.37	0.4	0.37	0.4	0.43
$\eta_{r,T}$ (St. Dev.—MP Shock)	0.22	0.24	0.27	0.23	0.25	0.27	0.23	0.25	0.28
$\eta_{r,E}$ (St. Dev.—MP Shock at ELB)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
$\eta^{SR}_{\pi,exp}$ (St. Dev.—SR Inflation Exp.)	0.09	0.11	0.12	0.08	0.1	0.11	0.09	0.1	0.12
$\eta^{LR}_{\pi,exp}$ (St. Dev.—LR Inflation Exp.)	0.07	0.08	0.09	0.09	0.11	0.12	0.11	0.13	0.14
Φ_1 (MP Switching—Taylor to ELB)	0.06	0.1	0.15	0.07	0.11	0.16	0.06	0.1	0.16
Φ_2 (MP Switching—ELB to Taylor)	0.07	0.16	0.31	0.06	0.12	0.29	0.07	0.16	0.35
γ (Constant Gain)	0.0584	0.0585	0.0587	0.059	0.0593	0.0602	0.0588	0.0589	0.0589
ω (Memory)	0.49	0.62	0.74	0.58	0.68	0.77	0.45	0.66	0.88
χ (Intensity of Choice)	0.38	0.51	0.66	1.12	1.47	1.96	0.24	0.36	0.46
Marg. (log-) Likl.		–410.05			–280.03			–333.92	

Appendix G. Robustness: Estimations without Inflation Expectations

This appendix presents the estimation results of the models in Section 3 without using inflation expectations data. The goal is to check the sensitivity of parameter estimates to inflation expectations data, and to examine whether the relative fitness of the models change when data on inflation expectations is excluded. The results are presented in Table G.1. In terms of model fitness, the relative ranking of the models remains the same as in Section 3. The HE model provides the best fit, followed by the AL model, the RE model with switching in MP, and finally the baseline RE model. This shows that the HE model improves model fitness not only along the margin of inflation expectations data but also on aggregate macrovariables. Parameters related to learning and heterogeneous expectations are all sensitive to expectations data: The estimated constant gain γ is higher in both the HE and the AL model when inflation expectations are included. The estimated memory in heterogeneous switching ω, as well as the intensity of choice χ, are both lower when estimated with expectations. These results show that including expectations data in the data set plays an important role in identifying parameters related to the learning process. It is also important to note that the results in this section are consistent with previous studies in the literature that have estimated learning and heterogeneous expectation models without using any survey data, e.g., Milani (2007), Slobodyan and Wouters (2012a, 2012b), and Cornea-Madeira, Hommes, and Massaro (2019), to name a few.

Table G.1. Posterior Distribution Moments for All Models, Estimated with Data on Only Inflation, Output Gap, and Nominal Interest Rate over the Period 1960:Q1–2019:Q4

	RE			RE—Switching in MP			Hetero			Learning		
Parameter	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**	**LB**	**Mean**	**UB**
$\overline{\pi}$ (Inflation Trend)	0.71	0.92	1.12	0.74	1.11	1.31	0.29	0.44	0.73	0.28	0.6	1.06
$\bar{r_T}$ (Int. Rate Trend—Taylor)	0.84	1.21	1.57	0.74	1.3	1.6	0.2	0.39	1.06	0.4	0.89	1.48
$\bar{r_E}$ (Int. Rate Trend—ELB)				0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.04
κ (NKPC Slope)	0.0066	0.0135	0.0238	0.0037	0.0086	0.0158	0.0125	0.0214	0.0276	0.0472	0.0571	0.0704
τ (Risk Aversion)	1.49	2.04	2.81	2.12	2.59	3.31	1.72	2.29	3.05	1.7	2.28	3.08
ι_y (Indexation—IS Curve)	0.09	0.15	0.21	0.08	0.14	0.19	0.05	0.07	0.13	0.11	0.16	0.22
ι_p (Indexation—NKPC)	0.03	0.07	0.13	0.03	0.07	0.11	0.02	0.04	0.09	0.02	0.04	0.09
ϕ_π (MP Inflation Reaction)	1.37	1.66	2	1.3	1.56	1.83	1.28	1.64	1.97	1.08	1.35	1.64
ϕ_y (MP Output Gap Reaction)	0.17	0.25	0.36	0.18	0.26	0.35	0.25	0.36	0.43	0.18	0.28	0.42
$\phi_{\Delta y}$ (MP Output Gap Growth Reaction)	0.08	0.1	0.13	0.07	0.08	0.1	0.07	0.1	0.13	0.04	0.06	0.08
ρ_r (MP Smoothing)	0.88	0.9	0.93	0.88	0.9	0.92	0.89	0.92	0.94	0.9	0.93	0.95
ρ_y (Persistence—Demand Shock)	0.86	0.91	0.95	0.83	0.88	0.92	0.88	0.93	0.95	0.53	0.61	0.68
ρ_π (Persistence—Supply Shock)	0.66	0.75	0.82	0.68	0.76	0.82	0.68	0.8	0.86	0.63	0.72	0.8
η_y (St. Dev.—Demand Shock)	0.15	0.2	0.26	0.16	0.21	0.27	0.18	0.21	0.27	0.33	0.38	0.46
η_π (St. Dev.—Supply Shock)	0.1	0.14	0.17	0.1	0.13	0.17	0.13	0.19	0.23	0.22	0.25	0.27
$\eta_{r,T}$ (St. Dev.—MP Shock)	0.2	0.22	0.24	0.2	0.22	0.24	0.2	0.22	0.25	0.19	0.21	0.22
$\eta_{r,E}$ (St. Dev.—MP Shock at ELB)				0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Φ_1 (MP Switching—Taylor to ELB)							0.07	0.11	0.19	0.1	0.18	0.33
Φ_2 (MP Switching—ELB to Taylor)							0.4	0.11	0.27	0.03	0.05	0.13
γ (Constant Gain)							0.0017	0.0036	0.0137	0.0062	0.0081	0.0105
ω (Memory)							0.67	0.78	0.94	0.57	0.76	0.9
χ (Intensity of Choice)							1.36	4.63	7.45	2.3	4.79	8.39
$1-q^T$ (Exog. Exit Probability—Taylor)				0.16	0.23	0.34						
$1-q^E$ (Exog. Exit Probability—ELB)				0.0028	0.012	0.03						
Marg. (log-) Likl.		–551.22			–527.88			–513.48			–516.69	
Bayes Factor		1			10.13			16.39			14.99	

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