Endogenous Growth, Skill Obsolescence, and Optimal Monetary Policy*

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We analyze Ramsey optimal monetary policy in a New Keynesian model with skill loss from long-term unemployment and endogenous growth through learning-by-doing. The competitive equilibrium is shown to be inefficient, despite imposing the Hosios condition, due to firms failing to internalize the effects of current hiring on (i) future labor productivity through learning-by-doing; and (ii) future training costs of other firms. These externalities are complementary to each other, thereby justifying marked deviations from price stability. In a calibrated version of the full model, we show significant deviations of the optimal policy from constant inflation, and from Taylor-type rules, in response to productivity shocks.

JEL Codes: E24, E52.

1. Introduction

The past two recessions have been challenging for macroeconomic stabilization policy. Both recessions featured an unusually strong increase in the share of long-term unemployed (LTU) workers. As Figure 1 illustrates, the share of LTU workers in the U.S. peaked at about 45 percent during the Great Recession and during the

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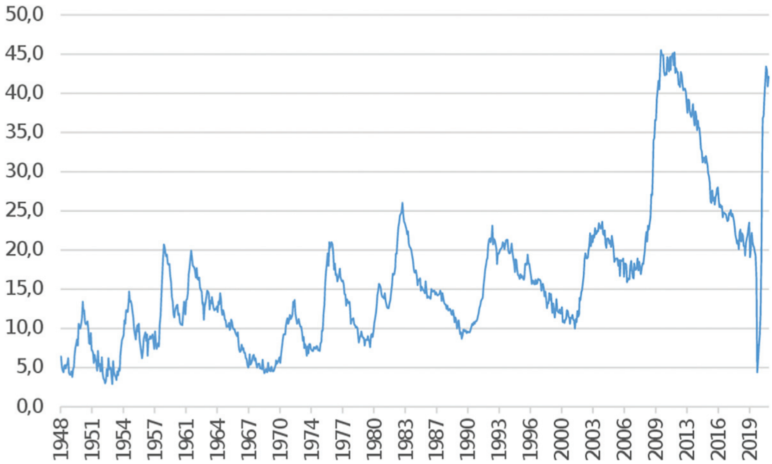
Figure 1. The Share of Long-Term Unemployment in Total Unemployment in the U.S.



Source: FRED.

COVID-19 pandemic, while in previous recessions the share of LTU workers never exceeded 25 percent. Furthermore, while the long-run effects of the COVID-19 pandemic are still unclear, many commentators agree that the Great Recession had permanent negative effects for GDP. This has led to a renewed interest in the hysteresis effects of deep recessions (see Cerra, Fatas, and Saxena 2020 for a recent survey). The implications of these two phenomena for macroeconomic stabilization policy are still underexplored.

In this paper, we analyze the role of endogenous growth and skill loss from long-term unemployment in optimal monetary policy. We do this in a tractable way, by introducing training costs associated with skill upgrading and learning-by-doing externalities into a New Keynesian model with search and matching frictions. Importantly, these two features of the model are complementary in the sense that their joint effects are larger than the sum of their individual effects. Our model is based on Lechthaler and Tesfaselassie (2023), who show that the model can account for key features of the Great Recession: (i) the "productivity puzzle"—the permanent gap between productivity and output relative to precrisis trends; and

(ii) the "missing disinflation puzzle"—the relative stability of inflation despite the pronounced fall in output. For this purpose, Lechthaler and Tesfaselassie (2023) assume a simple Taylor-type policy rule, as is standard in the literature. In the present paper, by contrast, we focus on Ramsey optimal monetary policy in the framework of that model.

Our analysis of optimal monetary policy proceeds in two stages. First, we consider a simple two-period model, and show the inefficiency of the competitive equilibrium analytically by comparing it to the outcome under the planner's problem. We identify two externalities: i) firms do not internalize the effects that hiring has on labor productivity through learning-by-doing; and ii) firms do not fully internalize the effects that hiring has on future training costs. The importance of the latter externality depends on the degree to which training costs are reflected in negotiated wages. Interestingly, even if wage bargaining is efficient, the externality still matters because future employers are not represented in the bargaining process.

Second, in a calibrated version of the full, infinite-horizon model, we illustrate our results quantitatively and conduct sensitivity analysis. Using impulse response functions, we show that the Ramsey optimal policy deviates from price stability so as to reduce inefficient fluctuations in response to a temporary productivity shock. Moreover, we analyze deviations of the Ramsey optimal policy from simple, Taylor-type rules considered in the related literature (e.g., Blanchard and Galí 2010). Here, we show that endogenous growth matters primarily for the medium- to long-run dynamics of output and human capital, because of the hysteresis effects it implies. By contrast, training costs matter primarily for the short-run and medium-run dynamics. Interestingly, the hysteresis effects are much smaller when the training cost is absent, which suggests the presence of complementarity between the endogenous growth channel and the training cost channel. We also analyze the sensitivity of optimal policy to the degree of sunkness in training costs and the strength of the learning-by-doing externality from aggregate employment to human capital accumulation and productivity growth. We show that optimal inflation volatility is lower the lower the degree of sunkness in training costs and the weaker the positive externality from aggregate employment to productivity growth.

Related Literature. A number of studies examine Ramsey optimal monetary policy in the presence of frictions in the labor market. The main finding in these studies is that optimal monetary policy deviates from price stability in response to inefficient employment fluctuations implied by labor market distortions. Faia (2009) shows the deviation from price stability when relaxing the Hosios condition for efficiency of the competitive equilibrium, which is that workers' bargaining power should equal the elasticity of the matching function with respect to unemployment. Thomas (2008) derives a quadratic approximation of the welfare function around a nondistorted steady state in a search and matching model with real-wage staggering and convex costs of posting vacancies, which generate monetary trade-offs. Ravenna and Walsh (2011) also derive a quadratic approximation of household welfare under flexible real wages and rationalize monetary trade-offs by assuming stochastic fluctuations in worker-firm bargaining shares. Faia, Lechthaler, and Merkl (2014) study optimal monetary policy in a labor selection model and show that optimal inflaton volatility rises with firing costs. Lechthaler and Snower (2013) study optimal monetary policy in a model with quadratic employment adjustment costs, where these costs depend on aggregate employment, thereby implying externalities in hiring decisions. Lechthaler and Tesfaselassie (2019) extend the search and matching model to include exogenous productivity growth and show that higher productivity growth exacerbates the effects of labor market distortions, thus calling for larger deviations from price stability. None of these papers consider endogenous growth and/or skill loss through long-term unemployment as we do. Closer to our paper, Annicchiarico and Rossi (2013) consider optimal monetary policy in the presence of learning-by-doing but within the standard New Keynesian model with competitive labor markets. Our analysis allows us to consider not only the role of labor market frictions and skill loss but also the complementarity between endogenous growth and skill loss.

The positive and normative implications of sunk costs have been studied within the labor search literature but, to our knowledge, not within the optimal monetary policy and business cycle literature, and not in the presence of endogenous growth. For instance, Acemoglu and Shimer (1999) study the efficiency of the search and matching model under the assumption that a firm makes ex ante investments before matching with a worker. They show that there is inefficiency provided investment costs are sunk, and the inefficiency can only be prevented by removing all the bargaining power from the worker. Cheron (2005) shows that when fixed match-specific costs are not sunk, the Hosios condition guarantees efficiency of the decentralized economy. Miyamoto (2011) finds similar results in the case where match-specific costs are endogenously determined. Pissarides (2009) introduces sunk matching costs as an amplification mechanism within the search and matching model, with the purpose of matching key labor market facts (in particular the volatility of unemployment). A paper closely related to ours is Acharya et al. (2022). Like us, they assume skill loss from unemployment that leads to the need for costly retraining. In combination with downward wage rigidity and the zero-lower-bound constraint, this can lead to a permanent unemployment trap. However, in contrast to our paper, they do not consider endogenous growth. Whereas the endogenous growth feature in our model leads to permanent output effects of temporary shocks, unlike Acharya et al. (2022), we follow the standard literature in focusing on local dynamics around a unique steady-state unemployment rate.[1]

There exists a small body of theoretical work that examines the relation between business cycle persistence and long-run output in the presence of endogenous growth. Chang, Gomes, and Schorfheide (2002) use learning-by-doing as a propagation mechanism in a real business cycle model. In their model, an increase in the number of hours worked contributes to future improvements in labor skills. Stadler (1990) compares the properties of real and monetary business cycle models in the presence of endogenous growth arising from learning-by-doing. A temporary shock is shown to induce a permanent upward shift in the aggregate production function, thus having long-run effects. Engler and Tervala (2018) use a two-country New Keynesian model to show that the fiscal output multiplier is significantly larger in the presence of learning-by-doing. Jordà, Singh, and Taylor (2020) demonstrate that monetary policy shocks can have long-lasting effects on productivity and output. Reifschneider,

[1]Our approach is consistent with the return of unemployment to precrisis level after a sharp and persistent rise during the Great Recession. In the calibrated version of the model, we set the steady-state unemployment rate by targeting the long-run average unemployment rate in the U.S.

Wascher, and Wilcox (2015) modify the large-scale Federal Reserve workhorse FRB/US macro model to allow for hysteresis effects and examine optimal monetary policy when policymakers face multiple concerns—financial stability, inflation stability, and unemployment hysteresis. They impose an ad hoc, nonlinear relationship between the natural rate of unemployment and the actual unemployment rate and do not consider Ramsey optimal policies. Garga and Singh (2021) also study optimal monetary policy in the presence of output hysteresis, deriving a quadratic approximation to welfare. They find that a monetary rule targeting output hysteresis is optimal at the zero lower bound. Unlike our paper, all these models abstract from labor market frictions and/or Ramsey optimal monetary policy.

The rest of the paper proceeds as follows. In Section 2 we present the simple, two-period model and analytically show the inefficiency of the decentralized economy in the presence of the two externalities of the model. In Section 3 we present the full, infinite-horizon model. In Section 4 we derive the objective of the Ramsey planner in the infinite-horizon model. Section 5 discusses the calibration of the model and presents the main results. Section 6 shows the relationship between the two externalities and optimal inflation volatility. Section 7 concludes.

2. A Two-Period Model

2.1 General Setup

In this section we develop a simple model with search and matching frictions, endogenous growth through learning-by-doing, and skill loss through long-term unemployment and demonstrate that this model features two externalities: i) private firms do not internalize the effects of their hiring decisions on human capital growth; and ii) private firms do not (fully) internalize the effects of these decisions on future training costs. These two externalities imply that even the economy without rigid prices and without monopolistic distortion is inefficient, giving the Ramsey planner a motive to deviate from price stability.[2]

[2]If the economy with flexible prices was efficient and the monopolistic distortion could be offset by a subsidy, the Ramsey planner would find it optimal to mimic the economy with flexible prices by holding the price level constant.

To be able to show these two externalities analytically and to develop the intuition behind these results, we use a model with the minimal structure that allows us to develop these insights. Most importantly, we restrict the model to two periods, which has the advantage of simplifying the wage bargaining process substantially. After having developed these insights we will use a full-fledged model for numerical analysis in Section 3.

Workers are assumed to be risk-neutral and live for two periods. The mass of workers is normalized to unity. The labor market is characterized by search and matching frictions whereby firms must pay a vacancy posting cost equal to κ. In the first period all workers start unemployed but have sufficient human capital and thus do not need training. All jobs last for only one period so that the number of searchers in both periods is unity. Those workers that find a job in the first period maintain their human capital and in aggregate generate endogenous growth in productivity through learning-by-doing. However, after production takes place all employed workers lose their job and start looking for a new job in period 2 (i.e., the separation rate is unity). Those workers that do not find a job in the first period lose their human capital and need training in the second period before production takes place (if they find a job).

The number of matches, M_t, $t \in \{1, 2\}$, is determined by a constant returns-to-scale matching function, with the number of searching workers, S_t, and the number of posted vacancies, V_t, as its arguments:

$$M_t = \mu S_t^{\alpha} V_t^{1-\alpha} = \mu V_t^{1-\alpha}, \tag{1}$$

where $\mu > 0$ is a scale parameter describing the efficiency of the labor market and $\alpha > 0$ is the elasticity of the matching function. The second equality follows from our assumption that all workers have to search for a job in both periods, i.e., $S_t = 1$. Dividing Equation (1) by V_t and defining labor market tightness as $\theta_t \equiv V_t/S_t$, we can write the vacancy-filling rate as

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \mu\theta_t^{-\alpha}. \tag{2}$$

Similarly, the job-finding rate is given by $\theta_t q(\theta_t)$.

Learning-by-doing as a driver of endogenous growth is introduced in a standard way: higher aggregate employment, N_t, generates a positive externality on the accumulation of aggregate human capital, H_{t+1} (due to enhanced opportunities for learning-by-doing). To capture this phenomenon, aggregate human capital in period 2 is given by

$$H_2 = BN_1H_1 \equiv h(N_1, H_1), \tag{3}$$

where $B > 0$ is a scale parameter, and H_1 is exogenously given.[3]

If in period 2 a firm is matched with a worker that was unemployed in period 1, the firm needs to upgrade the matched worker's skill at a cost of χ. The expected training cost in period 2 per hired worker, TC_2, is thus an increasing function in the share of period 1 unemployed, $u_1 = 1 - N_1$, in total job searchers in period 2,

$$TC_2 = \frac{u_1}{S_2}\chi = u_1\chi. \tag{4}$$

There is a risk-neutral representative household with a continuum of employed and unemployed workers. Unemployed workers receive unemployment benefits equal to u_b.[4] As is common in the literature, we assume that income is pooled within the household so that consumption is equalized across employed and unemployed members. Each period the household consumes all income. It discounts future consumption by the subjective discount factor, β.

2.2 Decentralized Economy

This section illustrates the solution of the decentralized, competitive economy in which wages are determined via Nash bargaining. The model is solved backwards, starting with period 2.

[3]As is common in the endogenous growth literature, the change in human capital is linear in the level of human capital. It is the absence of diminishing returns in human capital accumulation that allows the model to generate sustained growth.

[4]As is standard in the literature, unemployment benefits are assumed to be financed by a lump-sum tax and the government follows a balanced budget.

Second Period. Since in period 2 we do not need to take account of future values, the firm values are simply given by the following static equations:

$$J_2^S = H_2 - w_2^S \text{ and } J_2^L = H_2 - w_2^L,$$

where J^S and J^L denote, respectively, gross firm value from hiring short-term and long-term unemployed workers, while w^S and w^L are the corresponding wages. The price of output is normalized to unity and the output/productivity of workers is solely determined by their human capital, which is H_2. The human capital of a worker who was unemployed in period 1 is also H_2 because the training upgrades the worker's skills before production takes place. The value of hiring a long-term unemployed, net of the training cost χ, is then $J_2^L - \chi$.

The corresponding worker values are

$$W_2^S = w_2^S, \quad W_2^L = w_2^L \text{ and } U_2 = u_b,$$

where the value of unemployment U_2 is equal across workers, i.e., independent of employment in period 1.

Wages are set according to Nash bargaining so that the optimal surplus-sharing rule for matches with the short-term unemployed (respectively, long-term unemployed) is given by $W_2^S - U_2 = (1-\nu)/\nu J_2^S$ (respectively, $W_2^L - U_2 = (1-\nu)/\nu(J_2^L - \xi\chi)$), where ν is the bargaining power of the firm. The parameter ξ governs the extent to which training costs are sunk at the time of wage bargaining with an LTU worker. When $\xi = 0$, training costs are fully sunk and not at all reflected in the negotiated wage. By contrast, when $\xi = 1$ training costs are not sunk at all and are fully reflected in the negotiated wage (and borne by the worker according to their bargaining power). Combining the surplus-sharing rules, the job values and worker values gives the wage rules

$$w_2^S = \nu u_b + (1-\nu) H_2 \text{ and } w_2^L = \nu u_b + (1-\nu)(H_2 - \xi\chi).$$

The bargained wages are a weighted average of unemployment benefits and period-2 productivity, net of training costs that are part of the wage bargaining with the LTU worker. Here, the main advantage of the two-period structure is that future values do not enter the wage bargaining, implying simpler wage equations.

Free entry of firms implies that the value of a vacancy is zero. This implies the following vacancy creation condition for period 2:

$$\frac{\kappa}{q(\theta_2)} = (1 - u_1)\, J_2^S + u_1(J_2^L - \chi). \tag{5}$$

Substituting out the value functions, Equation (5) can be rewritten as

$$\frac{\kappa}{q(\theta_2)} + u_1\chi\,(1 - (1 - \nu)\,\xi) = \nu\,(H_2 - u_b)\,; \tag{6}$$

that is, the expected cost of hiring a worker, including the firm's share of the expected training cost, equals the expected firm profit generated by the worker.

First Period. In period 1 all workers have equal human capital, so there is only one wage level, one equation defining firm value J_1, and two equations defining worker values U_1 and W_1:

$$J_1 = H_1 - w_1,$$
$$U_1 = u_b + \beta\left(\theta_2 q(\theta_2) w_2^L + (1 - \theta_2 q(\theta_2))\, u_b\right),$$
$$W_1 = w_1 + \beta\left(\theta_2 q(\theta_2) w_2^S + (1 - \theta_2 q(\theta_2))\, u_b\right).$$

The value of a firm is just given by its contemporaneous profits. There is no continuation value because all jobs are destroyed at the end of period 1. For workers the continuation values depend on the employment status, because unemployed workers need retraining in period 2 if they are to become productive, implying a lower wage if training costs are shared.

As before, the surplus-sharing rule is $W_1 - U_1 = [(1 - \nu)/\nu]J_1$, so that the bargained wage is given by

$$w_1 = \nu\left[u_b - \beta\theta_2 q(\theta_2)\,(1 - \nu)\,\xi\chi\right] + (1 - \nu)H_1.$$

Note that, unless $\xi = 0$ (training costs are sunk), the threat point of a worker is lower than the unemployment benefit u_b. Thus, by accepting a lower wage, the worker compensates the firm for the benefit that current employment eliminates the need to pay training costs in period 2.

Optimal vacancy posting for period 1 is then

$$\frac{\kappa}{q(\theta_1)} = J_1 = \nu\left[H_1 - u_b + \beta\theta_2 q(\theta_2)\,(1 - \nu)\,\xi\chi\right]. \tag{7}$$

2.3 Planner Economy

The social planner chooses the number of vacancies in both periods, V_1 and V_2, so as to maximize the discounted sum of consumption in period 1 and period 2, $C_1^* + \beta C_2^*$ (where the superscript * indicates the planner economy). In period 1, consumption is equal to output less vacancy creation costs, $C_1^* = \mu {V_1^*}^{1-\alpha} H_1 - \kappa V_1^*$. In period 2, consumption is equal to output less vacancy creation costs and training costs, $C_2^* = \mu {V_2^*}^{1-\alpha} H_2^* - \kappa V_2^* - \left(1 - \mu {V_1^*}^{1-\alpha}\right) \mu {V_2^*}^{1-\alpha} \chi$. The first-order conditions with respect to V_1^* and V_2^* are, respectively,

$$\frac{\kappa}{(1-\alpha)\,\mu {V_1^*}^{-\alpha}} = H_1 + \beta\mu {V_2^*}^{1-\alpha}\chi + \beta B H_1 \mu {V_2^*}^{1-\alpha}, \text{ and} \tag{8}$$

$$\frac{\kappa}{(1-\alpha)\,\mu {V_2^*}^{-\alpha}} + u_1^*\chi = H_2^*. \tag{9}$$

In both Equations (8) and (9), the left-hand side is the expected cost of hiring a worker. In period 1, this is simply the expected vacancy posting cost, whereas in period 2 it also includes the expected training cost. Note that the planner takes account of the congestion externality by premultiplying the worker-finding rate with $(1-\alpha)$. On the right-hand side we see the benefits of hiring, which in period 2 is simply given by output. In period 1 the benefit includes two additional terms: the increase in period 2 productivity (the learning-by-doing effect, last term on the right-hand side) and the reduction in training costs in period 2 due to a smaller share of job searchers who lost human capital (the second term on the right-hand side).

2.4 Comparison

Comparing Equations (6) and (9), it can be seen that second-period vacancy posting in the decentralized economy is optimal if (i) the Hosios condition is satisfied ($\nu = 1-\alpha$); (ii) unemployment benefits are zero ($u_b = 0$); (iii) there is full sharing of training costs ($\xi = 1$); and (iv) unemployment in period 1 is optimal. The first two conditions are the well-known conditions for optimal vacancy posting in the standard search and matching model. The third condition extends this to the presence of training costs. It assures that training costs are shared between both parties in accordance with their shares of profits. If training costs were partly sunk (i.e., $\xi < 1$), firms would have to bear a disproportionate share of the training cost, leading to the creation of too few vacancies.

Using conditions (i)–(iii) in the vacancy posting condition for period 1, Equation (7), we get

$$\frac{\kappa}{q(\theta_1)} = (1-\alpha)\,H_1 + \alpha\beta\,(1-\alpha)\,\mu V_2^{1-\alpha}\chi, \tag{10}$$

where we replaced the job-finding rate $\theta_2 q(\theta_2)$ with $\mu V_2^{1-\alpha}$. There are two differences compared to the planner's solution (8). First, unlike the planner, firms in the competitive economy do not internalize the effect of vacancy creation in period 1 on productivity in period 2. Second, private firms internalize only partly the effect of hiring in period 1 on training costs in period 2. In other words, firms are not fully compensated for the positive externality that job creation maintains the human capital of workers, reducing the need for training in the future.

To see the training cost effect more clearly, assume $B = 0$, in which case the planner's optimality condition (8) simplifies to

$$\frac{\kappa}{\mu V_1^{-\alpha}} = (1-\alpha)\,H_1 + \beta\,(1-\alpha)\,\mu V_2^{1-\alpha}\chi. \tag{11}$$

This equation is very similar to the competitive equilibrium—Equation (10)—but note that the second term on the right-hand side of Equation (10) is multiplied by α, while this is not the case in Equation (11). A private firm partially internalizes the reduction in the training cost, because it benefits the worker (as the worker does not need to pay training costs in the future) and it can participate in this gain through a reduced wage payment. However, it does not fully internalize the effect because part of the benefit of reduced training accrues to the future employer of the worker, which is not represented in period 1 bargaining.

Thus, both features, learning-by-doing and skill loss through long-term unemployment, introduce an inefficiency into the decentralized economy. Private firms post too few vacancies because they do not fully internalize the beneficial effects of posting vacancies. While we have demonstrated this using a simplified model, the same mechanisms are at play in the full model. The Ramsey planner thus has a motive to use monetary policy in response to business cycle shocks to reduce inefficient fluctuations.

2.5 Monetary Policy

To demonstrate the role of optimal monetary policy in this simple framework, we introduce a monopolistically competitive final goods sector, which uses as input the homogeneous intermediate good produced by the sector with labor market frictions, as outlined above. This setup is similar to the full model developed further below.

Let the nominal price of the intermediate good be P_1^I. With a linear technology in the final goods sector, P_1^I represents the nominal marginal cost. Following Gabaix (2020), we assume that in period 1 a fraction ω of the final goods firms are unable to optimally set their prices, while in period 2 all prices are flexible. Let P_1 be the price level in the final goods sector, P_1^* the optimal price set by optimizing firms, and P_0 the predetermined price of non-optimizing firms. From this follows the standard price index $P_1 = \left(\int_0^1 P_{k,1}^{1-\epsilon} dk\right)^{\frac{1}{1-\epsilon}}$, where $\epsilon > 1$ controls the degree of monopolistic power. The price index can be rewritten as

$$1 = (1-\omega) p_1^{*(1-\epsilon)} + \omega \Pi_1^{\epsilon-1}, \tag{12}$$

where $p_1^* \equiv P_1^*/P_1$ is the optimal relative price and $\Pi_1 \equiv P_1/P_0$ is period-1 inflation. Since prices are flexible in period 2, only period-1 conditions affect P_1^*. In this case, $P_1^* = \mu_p P_1^I$, or $p_1^* = \mu_p p_1^I$, where $p_1^I \equiv P_1^I/P_1$ and $\mu_p \equiv \epsilon/(\epsilon-1)$ is the price markup.

In period 1, a firm in the intermediate goods sector with productivity H_1 receives nominal revenues of $P_1^I H_1$ and pays a nominal vacancy posting cost of $P_1 \kappa$. In period 2, the nominal training cost is $P_2 \chi$. Then, when written in nominal terms, the vacancy creation condition—Equation (10)—becomes

$$\frac{\kappa P_1}{q(\theta_1)} = (1-\alpha) P_1^I H_1 + \alpha\beta (1-\alpha) \mu V_2^{1-\alpha} P_2 \chi, \tag{13}$$

and in real terms,

$$\frac{\kappa}{q(\theta_1)} = (1-\alpha) p_1^I H_1 + \alpha\beta (1-\alpha) \mu V_2^{1-\alpha} \Pi_2 \chi. \tag{14}$$

Given that prices are rigid in period 1, monetary policy has real effects. In order to link monetary policy to job creation, we rewrite the price index (12) as

$$1 = (1-\omega)\mu_p \left(p_1^I\right)^{1-\epsilon} + \omega \Pi_1^{\epsilon-1}. \tag{15}$$

Together, Equations (14) and (15) show that variations in period-1 inflation are related to variations in the relative intermediate input price p_1^I and thus variations in job creation. Put differently, by moving the inflation rate, monetary policy can influence job creation. We have already shown above that job creation in the decentralized economy is inefficient due to externalities related to endogenous growth and training. Thus, it is optimal for monetary policy to use the channel described in Equations (14) and (15) to improve welfare. In general, the violation of the Hosios condition ($\nu \neq 1-\alpha$), nonzero unemployment benefits ($u_b > 0$), sunk training costs ($\xi < 1$), and learning-by-doing externalities ($B > 0$) matter for the degree of inefficiency in the competitive equilibrium, and thus for optimal monetary policy.

3. The Full Dynamic Model

We now present the baseline model—an infinite-horizon New Keynesian model featuring search and matching frictions, endogenous growth from learning-by-doing, and skill loss from long-term unemployment and associated training costs. Following the pioneering work of Walsh (2003), the model economy has two sectors: a retail sector and a wholesale sector. Firms in the wholesale sector combine raw labor and human capital to produce output and sell their output to the retail sector in a perfectly competitive market. The labor market is subject to search frictions that give rise to equilibrium unemployment. In contrast to Acharya et al. (2022), our model features a unique unemployment rate. As in the previous section, endogenous growth is generated by learning-by-doing and LTU workers need retraining.

Each retail firm transforms the wholesale good into a differentiated final good and sells it to households in a monopolistically competitive market. Retail firms set prices under Calvo-type nominal price staggering. Each household consists of a continuum of employed and unemployed (and searching) workers who pool their income.[5] Household utility depends on consumption only.

3.1 Labor Market and Human Capital Dynamics

We start by describing the aggregate relationships in the labor market within the wholesale sector and the endogeneity of aggregate human capital dynamics. As is standard, the size of the labor force is normalized to 1. At the beginning of each period, a fraction δ of previously employed workers are separated from their jobs. These unemployed workers immediately engage in job search. As a result, aggregate employment evolves according to the dynamic equation

$$N_t = (1-\delta)N_{t-1} + M_t, \tag{16}$$

where M_t is the number of newly formed matches in period t, which become productive immediately. Moreover, the number of searching workers in period t is given by

$$S_t = 1 - (1-\delta)N_{t-1}, \tag{17}$$

and the unemployment rate after hiring takes place is $u_t = 1 - N_t$.

The number of newly created matches, M_t, and the job-filling rate, $q(\theta_t)$, are given, respectively, by Equations (1) and (2), but for ease of reading we repeat them here:

$$M_t = \mu S_t^{\alpha} V_t^{1-\alpha}, \tag{18}$$

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \mu \theta_t^{-\alpha}. \tag{19}$$

The accumulation of aggregate human capital is generalized to

$$H_{t+1} = (1-\delta_H)H_t + BN_tH_t, \tag{20}$$

where δ_H is the depreciation rate of human capital. One can rewrite Equation (20) in terms of the gross growth rate of human capital,

$$\Gamma_{H,t+1} \equiv \frac{H_{t+1}}{H_t} = 1 - \delta_H + BN_t, \tag{21}$$

[5] As is well-known, locating labor market frictions and nominal price rigidities in different sectors as well as income pooling by workers make the model tractable.

which shows that a fall in aggregate employment today leads to a fall in future productivity growth.

3.2 Households

There is a representative household with a continuum of members over the unit interval. The period utility function is given by

$$U_t = \log C_t. \tag{22}$$

Household consumption, C_t, is a Dixit-Stiglitz composite of a continuum of differentiated goods, $C_t = \left(\int_0^1 C_{k,t}^{1/\mu_p} dk\right)^{\mu_p}$, where each good is indexed by k, $\mu_p = \frac{\epsilon}{\epsilon-1}$, and ϵ is the elasticity of substitution between goods. Optimal consumption allocation across goods gives the demand equation $C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\epsilon} C_t$, where $P_t = \left(\int_0^1 P_{k,t}^{1-\epsilon} dk\right)^{\frac{1}{1-\epsilon}}$ is the price index.

In any given period a fraction of household members, N_t, are employed by firms and earn a nominal wage, W_t. The rest earn nominal unemployment benefits of $P_t u_b H_t$, $u_b > 0$, and search for work.[6] As with the two-period model, income is pooled within the household so that consumption is equalized across employed and unemployed workers. The household maximizes the lifetime utility $E_t \sum_{i=0}^{\infty} \beta^i U_{t+i}$, where β is the subjective discount factor. The household's budget constraint is

$$P_t C_t + A_t = W_t N_t + P_t u_b H_t (1 - N_t) + R_{t-1} A_{t-1} + D_t, \tag{23}$$

where R_t is the nominal interest rate on bond holdings A_t, and D_t is aggregate nominal profit from ownership of retail firms.

It is straightforward to derive the familiar consumption Euler equation

$$1 = E_t \quad Q_{t,t+1} \frac{R_t}{\Pi_{t+1}} \quad , \tag{24}$$

[6]The presence of H_t ensures that along a balanced growth path (where aggregate variables such as output and consumption grow at the same rate as labor productivity), real unemployment benefits grow at the same rate as the real wage.

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate and $Q_{t,t+1} \equiv \beta\left(C_{t+1}/C_t\right)^{-1}$ is the household's stochastic discount factor.

3.3 Firms

Next, we describe the structure of the intermediate goods sector, followed by the final goods sector.

3.3.1 Intermediate Goods Sector

Intermediate goods firms can employ only one worker and produce with aggregate human capital H_t. The firms face standard search and matching frictions as well as frictions related to skill obsolescence and associated training costs incurred for skill upgrading.[7] There is an unlimited number of potential entrants that need to post a vacancy at real cost $H_t\kappa$ to have the chance to find a worker and enter the market. In addition, potential entrants expect to pay training costs if the matched worker needs a skill upgrade.

To introduce skill loss from long-term unemployment in a tractable way, a period is taken to represent six months.[8] Similar to Acharya et al. (2022), the long-term unemployed are those job seekers in period t whose last job was in period $t-2$ or earlier (in the U.S. this corresponds to those unemployed for 27 weeks or longer). By contrast, a searching worker in period t whose last job was in period $t-1$ does not need a skill upgrade. These two types of workers may be differentiated as long-term unemployed versus short-term unemployed. Consistent with these definitions, the expected training cost per hired worker, TC_t, is given by

$$TC_t = z_t\chi H_t, \tag{25}$$

where $z_t \equiv u_{t-1}/S_t$ is the ratio of the number of LTU job seekers to total job seekers. Thus, z_t is the probability that a firm matches with

[7] A detailed discussion of the standard search and matching model can be found in, e.g., Pissarides (2000).

[8] The main motivation for this assumption is that it simplifies the wage bargaining process because only two types of unemployed workers exist.

an LTU worker, and thus needs to upgrade the matched worker's skill at a cost of χH_t.[9]

It is important to note that the training cost is a predetermined endogenous variable (TC_t is given as of period t but responds to shocks with a one-period lag). An adverse shock in period $t-1$ that lowers employment, N_{t-1}, and the job-finding rate, $\theta_{t-1}q(\theta_{t-1})$, also increases the share of LTU workers in total job seekers in period t and thus the expected training cost, as given in Equation (25).

Let J_t^S (J_t^L) denote the value to a firm of matching with a short-term (long-term) unemployed worker. The value of a vacancy is then given by $q(\theta_t)\left[z_t\left(J_t^L - \chi H_t\right) + (1-z_t)J_t^S\right]$. Free entry of firms drives down the value of a vacancy to zero so that the vacancy creation condition, adjusted for the presence of a training cost and a balanced growth path, is

$$\kappa H_t = q(\theta_t)\left[z_t\left(J_t^L - \chi H_t\right) + (1-z_t)J_t^S\right]. \tag{26}$$

The cost of posting a vacancy equals the expected net benefit of posting a vacancy, the expected profits in case the search for a worker is successful. If the cost of posting a vacancy was lower than the expected profit of posting a vacancy, new vacancies would be posted, lowering the vacancy-filling rate and thereby expected profits until the incentive to post further vacancies vanishes. Likewise, an increase in the training cost has similar effects on the incentive to post vacancies. But crucially, the expected training cost depends on the probability that a new hire will come from the long-term unemployed, who need skill upgrading.

Active firms in this sector face a perfectly competitive output market. Let P_t^I denote the nominal market price and $p_t^I \equiv P_t^I/P_t$ the real market price. Then the value of a job filled with a short-term or long-term unemployed worker, respectively, is defined as

$$J_t^S = a_t H_t p_t^I - w_t^S + (1-\delta)E_t\left\{Q_{t,t+1}J_{t+1}\right\} \text{ or} \tag{27}$$

$$J_t^L = a_t H_t p_t^I - w_t^L + (1-\delta)E_t\left\{Q_{t,t+1}J_{t+1}\right\}, \tag{28}$$

[9] The presence of H_t ensures that along the balanced growth path the vacancy posting cost and the training cost grow at the same rate as aggregate labor productivity. Without the above assumption, over time vacancies would converge toward infinity and unemployment toward zero, since the ratio of vacancy creation costs to labor productivity would converge toward zero.

where a_t is productivity and $w_t^m = W_t^m/P_t$ is the real wage of worker type $m \in \{S, L\}$. The value of a firm is the sum of the current profits and the expected future value of the match discounted by the appropriate discount factor. Note that because of skill upgrading the continuation value of a match with an LTU worker is equal to that of a short-term unemployed worker.

In response to a positive productivity shock (i.e., higher a_t), firms post more vacancies. As in the standard search and matching model, the resulting increase in labor market tightness increases the average duration of vacancies, and thus raises the expected cost of vacancy creation. However, since training costs are predetermined here, the total expected cost of hiring does not increase in proportion to the decrease in the job-filling rate. Thus, the presence of sunk training costs has an amplification effect on vacancy creation and market tightness.

Note also that, as aggregate vacancies rise, the share of long-term unemployment in total unemployment falls. This implies that firms expect future training costs, tc_{t+1}, to decline. This effect alone reduces the continuation value of a match and thus lowers the incentive to post a vacancy.

The wage rate is set under the standard assumption of Nash bargaining. The real value of employment and unemployment to a continuing worker and a searcher who is short-term unemployed are the same. The real value of employment is given by

$$W_t^S = w_t^S + E_t\left\{Q_{t,t+1}\left[(1-\delta(1-f_{t+1}))W_{t+1}^S + \delta(1-f_{t+1})U_{t+1}\right]\right\}, \tag{29}$$

where $f_{t+1} \equiv \theta_{t+1}q(\theta_{t+1})$ is the job-finding rate.

The real value of employment to a worker who was long-term unemployed, i.e., unemployed as of period $t-1$ or earlier, is

$$W_t^L = w_t^L + E_t\left\{Q_{t,t+1}\left[(1-\delta(1-f_{t+1}))W_{t+1}^S + \delta(1-f_{t+1})U_{t+1}\right]\right\}. \tag{30}$$

The corresponding real value of unemployment to short-term and long-term unemployed is the same because both get the same unemployment benefit and will have the same level of skills next period. Thus,

$$U_t = u_b H_t + E_t\left\{Q_{t,t+1}\left[f_{t+1}W_{t+1}^L + (1-f_{t+1})U_{t+1}\right]\right\}. \tag{31}$$

Under Nash bargaining, the optimal surplus-sharing rule for new matches with the long-term unemployed (respectively, continuing workers or short-term unemployed) is given by $S_t^L = \bar{\nu}(J_t^L - \xi\chi H_t)$, (respectively, $S_t^S = \bar{\nu}J_t^S$), where $\bar{\nu} \equiv (1-\nu)/\nu$, and ν is the bargaining power of the firm. We have

$$\begin{aligned} S_t^S &= w_t^S - u_b H_t + E_t \quad Q_{t,t+1}[(1-\delta)(1-f_{t+1})S_{t+1}^S \\ &\quad + f_{t+1}(S_{t+1}^S - S_{t+1}^L)] \quad , \end{aligned} \tag{32}$$

$$\begin{aligned} S_t^L &= w_t^L - u_b H_t + E_t \quad Q_{t,t+1}[(1-\delta)(1-f_{t+1})S_{t+1}^S \\ &\quad + f_{t+1}(S_{t+1}^S - S_{t+1}^L)] \quad , \end{aligned} \tag{33}$$

so that $S_t^S - S_t^L = w_t^S - w_t^L$. Using this equation, the surplus-sharing rules, and the relation $J_t^S - J_t^L = -(w_t^S - w_t^L)$, we get $J_t^S - J_t^L = -(1-\nu)\xi\chi H_t$, $S_t^S - S_t^L = (1-\nu)\xi\chi H_t$, and $w_t^L = w_t^S - (1-\nu)\xi\chi H_t$. In the limiting case $\xi = 0$ (i.e., training costs are fully sunk) both types of workers earn the same wage. By contrast, when $\xi > 0$, the LTU workers receive lower wages because they bear part of the training costs. Moreover, the larger the value of ν, that is, the higher the bargaining power of the firm, the larger is the gap between the wages of workers who were long-term unemployed and short-term unemployed.

3.3.2 Final Goods Sector

Each firm in the final goods sector, k, produces a differentiated final good using a linear technology, $Y_{k,t} = Y_{k,t}^I$, and receives a subsidy, τ, so that the firm's real marginal cost, $mc_{k,t}$, is given by $(1-\tau)p_t^I$. Price setting is subject to Calvo-type price staggering, where ω is the fraction of firms whose prices are fixed in any given period. Let $P_{k,t}$ denote firm $k's$ output price. Each firm, k, maximizes lifetime profit, $E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \left(P_{k,t}/P_{t+i} - (1-\tau)p_{t+i}^I\right) Y_{k,t+i}$, subject to the total demand for good k, $Y_{k,t+i} = (P_{k,t}/P_{t+i})^{-\epsilon} Y_{t+i}$, where $Y_{t+i} = C_{t+i} + H_{t+i}\kappa V_{t+i} + \chi \frac{u_{t-1+i}}{S_{t+i}} q(\theta_{t+i})V_{t+i}$ is total aggregate demand that includes consumption and the vacancy posting costs and training costs. The resulting optimal price is

$$p_t^* = \mu_p \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} (1-\tau) p_{t+i}^I \frac{Y_{t+i}}{Y_t} \left(\frac{P_{t+i}}{P_t} \right)^{\epsilon}}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \frac{Y_{t+i}}{Y_t} \left(\frac{P_{t+i}}{P_t} \right)^{\epsilon-1}}, \tag{34}$$

where $p_t^* \equiv P_t^*/P_t$, $y_t = Y_t/H_t$, and μ_p is the price markup in the absence of price staggering. Endogenous growth feeds back into optimal pricing through two counteracting effects. Lower expected growth implies a lower discount rate (higher stochastic discount factor) but also lower expected future demand growth.

Equation (34) can be rewritten as

$$p_t^* = \mu_p \frac{F_{n,t}}{F_{d,t}}, \tag{35}$$

where $F_{n,t}$ and $F_{d,t}$ are auxiliary variables given by

$$F_{n,t} = (1-\tau) p_t^I y_t c_t^{-1} + \omega\beta \left(\frac{c_{t+1}}{c_t} \right)^{-1} \Pi_{t+1}^{\epsilon} F_{n,t+1}, \tag{36}$$

and

$$F_{d,t} = y_t c_t^{-1} + \omega\beta \left(\frac{c_{t+1}}{c_t} \right)^{-1} \Pi_{t+1}^{\epsilon-1} F_{d,t+1}. \tag{37}$$

Under Calvo-type price staggering, the aggregate price index can be rewritten as

$$1 = (1-\omega) p_t^{*(1-\epsilon)} + \omega \Pi_t^{\epsilon-1}. \tag{38}$$

Aggregating both sides of the market clearing condition for the intermediate good and using the demand equation for the final good k leads to a relationship between aggregate final output y_t and intermediate good output y_t^I,

$$y_t^I = \Delta_t y_t, \tag{39}$$

where $\Delta_t \equiv \int_0^1 \left(P_{k,t}/P_t \right)^{-\epsilon} df$ is a measure of price dispersion, which can be rewritten as

$$\Delta_t = (1-\omega) p_t^{*-\epsilon} + \omega \Pi_t^{\epsilon} \Delta_{t-1}. \tag{40}$$

As aggregate output in the intermediate goods sector is equal to aggregate employment, Equation (39) can be rewritten as

$$N_t = \Delta_t y_t. \tag{41}$$

Finally, the aggregate resource constraint in stationary form is given by

$$y_t = c_t + \kappa V_t + tc_t q(\theta_t) V_t. \tag{42}$$

4. Ramsey Optimal Monetary Policy

The Ramsey planner maximizes household utility subject to the competitive equilibrium under nominal price rigidity and labor market frictions, i.e., takes the distortions on the labor market as given. As is standard, we assume the government subsidy τ is set such that it eliminates monopolistic distortions in steady state.[10] We point out, however, that the government subsidy is not time-varying, and therefore monopolistic distortions might reappear in response to business cycle shocks.

We first transform the objective of the planner into a stationary form. This is necessary, because our model features positive long-run growth. The objective function as expressed in terms of the level of consumption is

$$\Pi_t = E_t \sum_{i=t} \beta^{i-t} \log C_i. \tag{43}$$

When this is reformulated in recursive form we get

$$\Pi_t = \log C_t + \beta E_t \Pi_{t+1}. \tag{44}$$

In Section A.1 of the appendix we show that maximizing (44) is equivalent to maximizing the detrended objective

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log \Gamma_{H,t+1} + \beta E_t \Pi'_{t+1} \tag{45}$$

[10] Thus we have the well-known condition that the optimal level of the subsidy rate τ is set equal to $1/\epsilon$.

Table 1. Assigned Parameters

Parameter		Value	Target/Source
β	Subjective Discount Factor	0.99^2	Standard Value
ω	Fraction of Non-Optimizing Firms	0.56	Blanchard and Galí (2010)
δ_H	Human Capital Depreciation Rate	0.0375	Jones, Manuelli, and Stacchetti (2000)
α	Elasticity (Matching Function)	0.5	Pissarides (2009)
ν	Firm's Share of Surplus	0.5	Hosios Condition
u_b	Unemployment Benefit Parameter	0.44	Replacement Rate of 0.5
ξ	Sunk Cost Parameter	0	Lechthaler and Tesfaselassie (2023)
ϕ_π	Inflation Coefficient	5	Blanchard and Galí (2010)
ϕ_u	Unemployment Coefficient	–0.8	Blanchard and Galí (2010)

expressed in normalized consumption c. The objective function (45) can be written alternatively in terms of employment by using the human capital accumulation equation to substitute out $\Gamma_{H,t+1}$,

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta}\log(1-\delta_H + BN_t) + \beta E_t \Pi'_{t+1}. \tag{46}$$

Interestingly, in the presence of endogenous growth ($B > 0$) the Ramsey planner's relevant welfare function depends directly on the level of aggregate employment N_t.

The economy under the Ramsey optimal policy is compared to a benchmark, simple Taylor-type rule considered in the related literature (e.g., Blanchard and Galí 2010):

$$R_t = R \left.\frac{\Pi_t}{\Pi}\right.^{\phi_\pi} \left(\frac{u_t}{u}\right)^{\phi_u}, \tag{47}$$

where the variables without time subscript denote steady-state values.

5. Calibration and Main Results

5.1 Calibration

Table 1 shows assigned parameters, while Table 2 shows the calibration of the model to the U.S. economy at a biannual frequency.

Table 2. Calibrated Parameters

Parameter		Value	Target/Source
δ	Job Separation Rate	0.2105	Share of Long-Term Unemployment 0.2
μ	Matching Efficiency	0.89	Job-Finding Rate 0.8
κ	Vacancy Posting Cost	0.4	Vacancy-Filling Rate 0.9
χ	Training Cost	0.6	Ratio of Training Costs to Vacancy Posting 0.3
B	Learning-by-Doing Coefficient	0.055	Steady-State Growth of 3 percent

Note: Following the related literature, we meet the respective target precisely by choosing the parameter under consideration.

The biannual values of Γ_H, δ_H, and ω, as well as the targeted steady-state job-filling rate, are based on the quarterly values used in Lechthaler and Tesfaselassie (2023). As in that paper, we target a steady-state growth rate of 3 percent. The steady-state unemployment rate is set at 5 percent and the job separation rate δ is set such that the implied share of long-term unemployment in total unemployment in the U.S. before 2008 is about 20 percent (Acharya et al. 2022). The implied steady-state job-finding rate is 0.8. The elasticity of the matching function α is set at 0.5, values that are common in the literature (see, e.g., Pissarides 2009). We impose the Hosios condition for efficiency in the absence of sunk training costs and learning-by-doing externalities, so that the firm's share of surplus ν is equal to the elasticity parameter in the matching function α. The scale parameter in the matching function μ and steady-state labor market tightness are set targeting the steady-state job-finding rate and the steady-state job-filling rate. The replacement rate is set at 0.5 (the implied value of the unemployment benefit parameter u_b is 0.44). This value is well within the range typically used in the literature. For instance, regarding the replacement rate that includes the value of leisure and home production, which we do not model, Shimer (2005), Hagedorn and Manovskii (2008), and Hall and Milgrom (2008) use, respectively, 0.4, 0.71, and 0.95.[11]

[11] In Section A.3 of the appendix, we show the sensitivity of optimal inflation volatility to the parameter u_b.

As a baseline the sunk cost parameter ξ is set at zero (Pissarides 2009; Acharya et al. 2022; Lechthaler and Tesfaselassie 2023), but we also undertake sensitivity analysis with respect to this parameter. We target a steady-state ratio of training costs to vacancy posting costs equal to 0.3, which is at the lower end of values considered in Pissarides (2009).[12] The training cost parameter χ and the cost of posting a vacancy κ are set consistent with the resulting steady-state solution of the model (see Table 2). The scale parameter in the human capital accumulation equation B is consistent with the steady-state annualized growth rate and steady-state employment rate. Regarding the Taylor rule coefficients ϕ_π and ϕ_u, we set these at 5 and -0.8 (the optimal simple rule in Blanchard and Galí 2010 under a U.S. labor market).

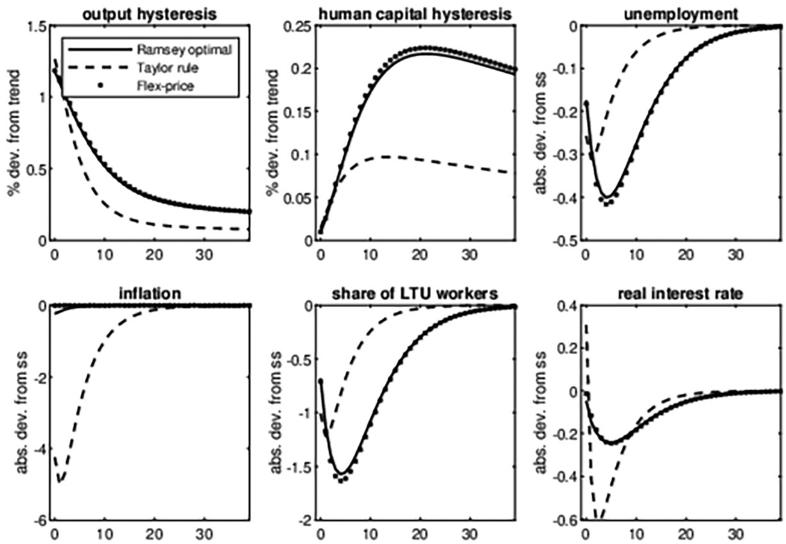
5.2 Impulse Responses

The quantitative analysis of the Ramsey optimal monetary policy is done using impulse responses to a temporary but persistent shock to productivity, a_t. To be specific, a_t is assumed to follow an autoregressive process of order 1: $a_t = a_{t-1}^{\rho_a} u_{at}$, $0 < \rho_a < 1$. In line with previous studies, the autocorrelation coefficient ρ_a is set equal to 0.9^2, while the standard deviation of the innovation u_{at} is set equal to 0.01.

Figure 2 shows the impulse responses of output, human capital, unemployment, the inflation rate, the share of LTU workers in total unemployment, and the real rate of interest to a positive productivity shock under the Ramsey optimal policy (solid line) and, for comparison, under a simple Taylor-type rule (dashed line) and under flexible prices (starred). Reflecting the underlying endogenous growth in the model, the impulse response named "output hysteresis" shows the gap between actual output and output in the absence of the shock, expressed as a percentage of the latter. The impulse response named "human capital hysteresis" is defined analogously. The impulse responses of inflation and the real interest rate are shown in absolute deviations and annualized.

[12]We think the chosen value is reasonable, as Pissarides (2009) considers fixed matching costs that may also include "costs of finding out about the qualities of the particular worker, of interviewing, and of negotiating with her."

Figure 2. Impulse Response to a Temporary Rise in Labor Productivity: Comparing Ramsey Optimal, Taylor Rule, and Flexible Price



A temporary increase in labor productivity raises output and human capital above their pre-shock trend and lowers unemployment, the share of long-term unemployment in total unemployment, and the inflation rate. The lower share of LTU workers implies a reduction in expected training costs that amplifies the drop in unemployment (see also Lechthaler and Tesfaselassie 2019). In turn, the presence of endogenous growth implies that the temporary shock to productivity has permanent effects on the level of human capital and output—the surge in employment enhances learning-by-doing and pushes the economy to a permanently higher level.

Under the policy that follows an unemployment-targeting Taylor rule, the effect of the productivity shock on real variables is much less pronounced than under Ramsey optimal policy. For output and human capital the differences are especially pronounced in the long run, where the more expansionary Ramsey policy implies much larger hysteresis effects. Under Ramsey policy, inflation declines by less and the real interest rate (reflecting the inertial nature of optimal policy; Woodford 2003) is less volatile than is the case under the Taylor rule. We can conclude that in the model under consideration the policy that follows an unemployment-targeting Taylor rule is suboptimal, implying too much volatility in inflation but too little volatility in output and unemployment.

The deviation from price stability under the Ramsey optimal policy also implies that the decentralized economy with flexible prices features excess volatility of unemployment and, importantly, an inefficiently strong response of human capital and output along the adjustment to their higher long-run levels.[13] In response, and in the presence of nominal price rigidity, the Ramsey planner uses the aggregate demand channel to reduce demand for the final good and in turn demand for the intermediate good. The resulting reduction in the relative price of the intermediate good p_t^I implies that the marginal revenue product of labor, and thus the match surplus, also decline. At the same time, a reduction in p_t^I implies a reduction in the real marginal cost of final good producers and therefore inflation.

To summarize, the economy with flexible prices exhibits excess volatility in output and unemployment, while the economy with rigid prices and a Taylor rule exhibits too little volatility. Optimal monetary policy lies between both cases but closer to the model with flexible prices.

5.3 Endogenous Growth Versus Training Costs

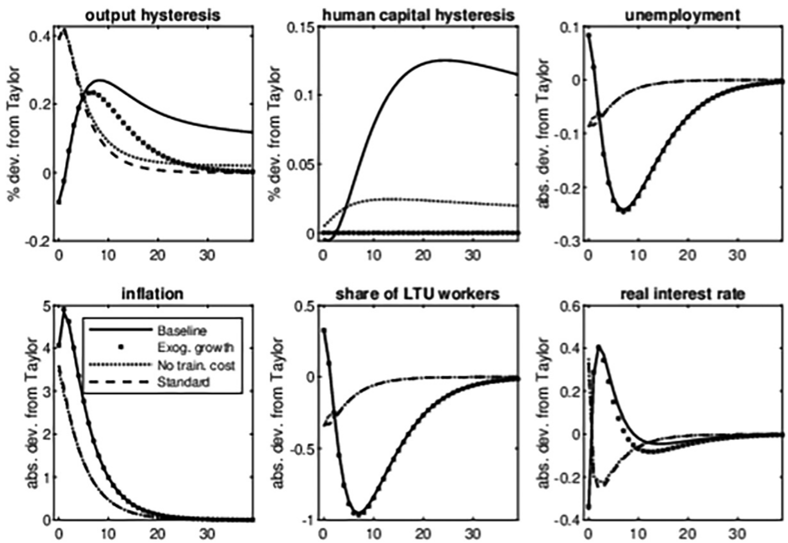
As noted above, our baseline model features two separate but closely related deviations from the standard model: endogenous growth based on human capital through learning-by-doing, and training costs related to the skill loss of LTU workers.[14] In order to see the role of each effect in isolation, Figure 3 shows the impulse responses of a model in which one or both of the two features are absent.

The starred line in Figure 3 shows impulse responses when only the training cost channel is present, the dotted line shows impulse

[13] If the decentralized economy with flexible prices were efficient, the Ramsey planner would keep inflation constant at zero.

[14] As shown in Lechthaler and Tesfaselassie (2023), both features are necessary to yield impulse responses that are broadly consistent with the recent empirical findings on the fall in productivity growth after deep recessions and the observed relative stability of inflation despite the pronounced fall in GDP during the Great Recession.

Figure 3. Impulse Response to a Temporary Rise in Labor Productivity: Comparing Baseline with Three Limiting Cases



Note: Limiting cases are: exogenous growth, no training costs, standard model. Each line shows the difference between the Ramsey optimal and the Taylor rule in the respective model.

responses when only the endogenous growth channel is present, and the dashed line shows impulse responses when both features are absent. In each case the impulse responses show the difference of a variable under Ramsey policy to that under Taylor rule policy. For instance, the solid line in the lower left panel shows that the Ramsey planner pushes up inflation by 4 percentage points relative to the economy under a Taylor rule, the difference between both lines in Figure 2. We show this difference so as to illustrate the effects of Ramsey policy in a compact way. Section A.2 of the appendix shows complementary figures that illustrate the full impulse responses (i.e., not the differences).

Three observations can be made from Figure 3. First, endogenous growth matters primarily for output and human capital in the medium to long run, because of the hysteresis effects it implies. In the short run, and for the other macroeconomic variables, endogenous growth has very little effect (starred and solid lines are almost identical).

Second, when the training cost channel is absent (dotted line), the Ramsey planner raises output much more in the short run, but less in the long run. The hysteresis effects are much smaller when the training cost is absent (dotted line vs solid line), which suggests the presence of complementarity between the endogenous growth channel and the training cost channel.

Third, looking at the dynamics of the real interest rate when the training cost channel is absent (dotted line), the fact that on impact the difference is positive is a result of the Taylor rule generating a more negative real interest rate than does the Ramsey optimal policy (by comparison, in the baseline case shown in Figure 1, on impact the real rate rises under the Taylor rule while it falls under the Ramsey optimal policy).

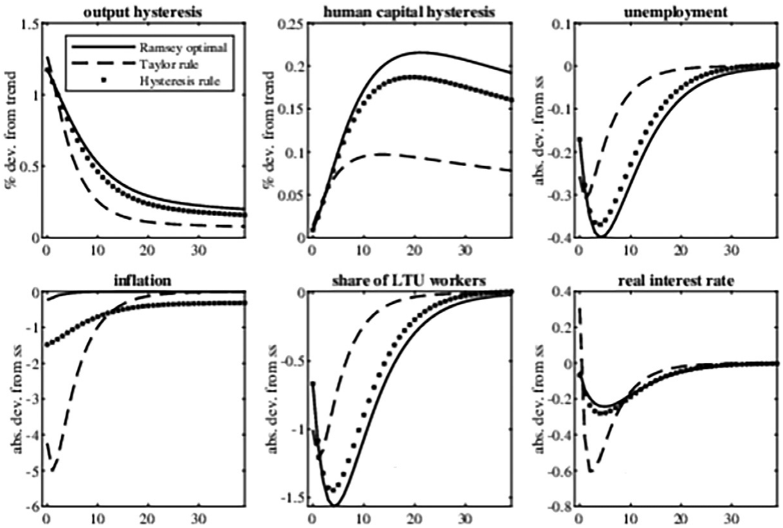
Finally, overall the standard model (dashed line) is very close to the model with endogenous growth but no training costs (dotted line). Thus, endogenous growth per se does not lead the Ramsey planner to deviate much from the policy pursued in the standard model.

5.4 Hysteresis Targeting Rule

For our baseline calibrations we used the Taylor-type rule given in Equation (47), suggested by Blanchard and Galí (2010). This rule has the advantage that it is not dependent on trend growth in productivity or output, since it is based on deviations of actual unemployment from steady-state unemployment, where the latter is still stationary in our model. Thus, the rule does not need to aim at a moving target. However, it is also clear from the discussion above that this rule is not optimal, and that other variations of the Taylor rule might be superior. Of special interest in our case are rules that take account of deviations from the previous long-run trend, rules that are able to target hysteresis effects.

Therefore, we show in this section the implications of using a monetary policy rule that takes account of past and present deviations in the growth rate of aggregate output from pre-shock trend.

Figure 4. Impulse Response to a Temporary Rise in Labor Productivity: Comparing Ramsey Optimal, Taylor Rule, and Hysteresis Rule

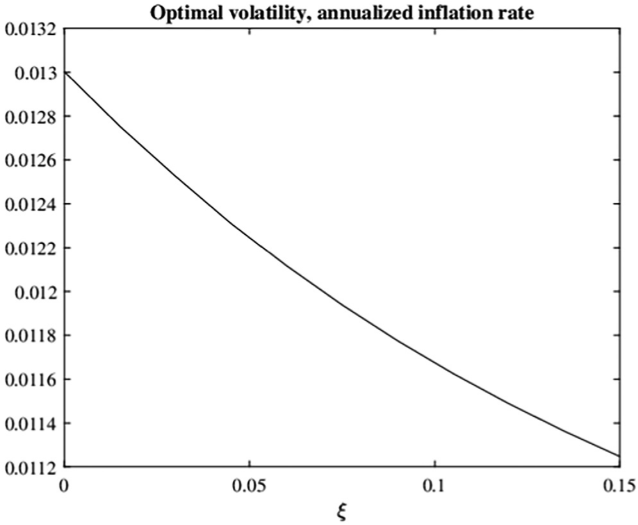


Specifically, we use a rule similar to the one suggested by Garga and Singh (2021),

$$R_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} (h_t)^{\phi_y}\,, \tag{48}$$

where $h_t = \Gamma_{y,t+1}/\Gamma_y * h_{t-1}$ is the cumulative sum of all deviations of output growth from output growth in the absence of shocks. Thus, this rule reacts to any deviations from previous trend, and so implies a return to the pre-shock trend. Figure 4 illustrates the results, comparing the hysteresis-targeting rule with the unemployment-targeting rule and Ramsey policy. It can be seen that the hysteresis-targeting rule implies much smaller swings in the real interest rate than the unemployment-targeting rule. In fact, the real interest rate under hysteresis targeting comes reasonably close to the one under Ramsey policy. By implication, human capital and output are higher than under the unemployment-targeting rule. It should be noted, however, that the hysteresis-targeting rule implies that output comes

Figure 5. Sunkness of Training Costs and the Optimal Volatility of Inflation



back to the pre-shock trend. This is not the case under Ramsey policy, under which output stays permanently above the pre-shock trend.

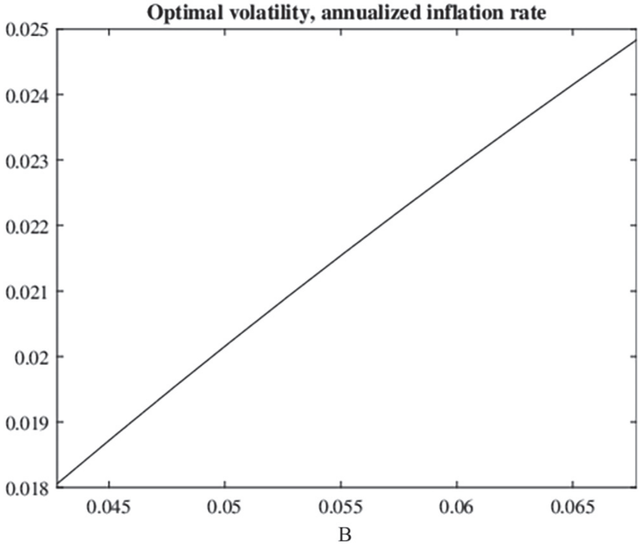
6. Optimal Inflation Volatility

The two key parameters of the model are the degree of sunkness of training costs (ξ) and the strength of the positive externality from aggregate employment to productivity growth (B).

We first show the relation between ξ and optimal inflation volatility when only skill loss from long-term unemployment is operative; that is, in the absence of endogenous growth ($B = 0$). Figure 5 shows the optimal inflation volatility as a function of the degree of sunkness of training costs (ξ) around the baseline case $\xi = 0$. The upper bound for ξ is chosen for computational reasons—it is set such that the steady-state employment rate remains below unity.

Optimal inflation volatility declines monotonically with ξ, which confirms our analytical results based on the two-period model of Section 2. Optimal volatility declines by about 13 percent as ξ increases from 0 to 0.15. The intuition is straightforward: the larger

Figure 6. Learning-by-Doing Externality and the Optimal Volatility of Inflation



the value of ξ (i.e., the lower the degree of sunkness in training costs), the more efficient wage bargaining, and thus the competitive equilibrium, becomes, implying that monetary policy focuses more on offsetting inefficiencies arising from nominal distortions (and in turn implying less deviation from price stability).

Next, we show the relation between B and optimal inflation volatility when only the learning-by-doing externality is operative; that is, in the absence of skill loss from long-term unemployment (no training costs are incurred). Figure 6 shows the optimal inflation volatility as a function of the strength of the learning-by-doing externality (controlled by the parameter B in Equation (20)) around the baseline calibration $B = 0.055$.

Optimal inflation volatility increases monotonically with B, which also confirms our analytical results based on the two-period model of Section 2. The intuition is again straightforward: the larger the value of B (i.e., the stronger the positive externality from aggregate employment to productivity growth), the less efficient the competitive equilibrium becomes, implying that monetary policy focuses less on offsetting inefficiencies arising from nominal distortions (and in turn implying greater deviations from price stability).

7. Concluding Remarks

We analyze Ramsey optimal monetary policy in a New Keynesian model with search and matching frictions featuring training costs due to skill loss from long-term unemployment and endogenous growth through learning-by-doing externalities. We show that the competitive equilibrium is inefficient due to two sources of externalities; that is, firms fail to internalize the effects that hiring has on labor productivity through learning-by-doing, and do not fully internalize the effects that hiring has on future training costs. These externalities lead to inefficient fluctuations, and their complementarity justifying marked deviations from price stability in response to productivity shocks. Optimal inflation volatility is shown to be increasing in the degree of sunkness of training costs and in the strength of the learning-by-doing externality.

While our framework focuses on skill loss from long-term unemployment and output hysteresis, one can extend the framework to allow for the presence of unemployment hysteresis, for example, by introducing labor force participation into the search and matching block of the model. Another possible extension is the incorporation of wage rigidity into the bargaining process and how this affects the degree to which firms internalize the effects that hiring has on future training costs. We leave this for future research.

Appendix

A.1 Detrended Model for Optimal Policy

The planner's problem is to maximize

$$\Pi_t = E_t \sum_{i=t} \beta^{i-t} \log C_i, \tag{A.1}$$

subject to the three constraints

$$N_t = (1-\delta)N_{t-1} + \mu\left(1-(1-\delta)N_{t-1}\right)^{\alpha} V_t^{1-\alpha}, \tag{A.2}$$

$$\begin{aligned} C_t = a_t H_t N_t - H_t \kappa V_t - H_t \chi \left[1 - \mu\left(1-(1-\delta)N_{t-2}\right)^{\alpha-1} V_{t-1}^{1-\alpha}\right] \\ \times (1-N_{t-2}) * \mu\left(1-(1-\delta)N_{t-1}\right)^{\alpha-1} V_t^{1-\alpha}, \end{aligned} \tag{A.3}$$

$$H_{t+1} = (1 - \delta_H + BN_t)\, H_t. \tag{A.4}$$

This is equivalent to maximizing

$$\Pi_t = E_t \sum_{i=t} \beta^{i-t} \left(\log c_i + \log H_i\right), \tag{A.5}$$

subject to the constraints

$$N_t = (1-\delta)N_{t-1} + \mu \left(1 - (1-\delta)N_{t-1}\right)^{\alpha} V_t^{1-\alpha}, \tag{A.6}$$

$$\begin{aligned} c_t = a_t N_t - \kappa V_t - \chi &\left[1 - \mu \left(1 - (1-\delta)N_{t-2}\right)^{\alpha-1} V_{t-1}^{1-\alpha}\right] \\ &\times \left(1 - N_{t-2}\right) * \mu \left(1 - (1-\delta)N_{t-1}\right)^{\alpha-1} V_t^{1-\alpha}, \end{aligned} \tag{A.7}$$

$$\Gamma_{H,t+1} = \left(1 - \delta_H + BN_t\right). \tag{A.8}$$

The problem is still not stationary because the object function contains H, which is not stationary. Thus, our goal is to split the objective into a part that is stationary (Π_t') and a remaining part that only depends on current H_t (which is the predetermined state variable as of period t). So let us use the transformation

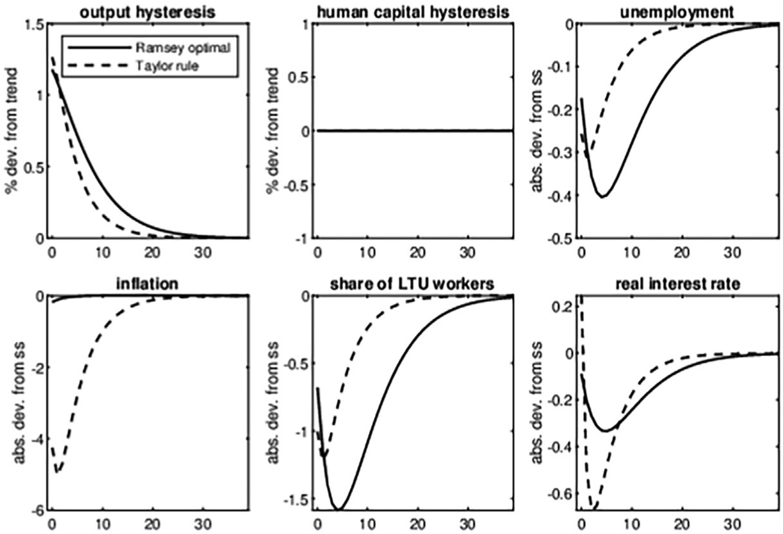
$$\Pi_t = \Pi_t' + \lambda \log H_t,$$

where the transforming factor λ has still to be determined such that it makes Π_t' stationary. Reformulating the above equation,

$$\begin{aligned} \Pi_t' &= \Pi_t - \lambda \log H_t \\ &= \log c_t + \log H_t - \lambda \log H_t + \beta \Pi_{t+1} \\ &= \log c_t + (1-\lambda) \log H_t + \beta\lambda \log H_t - \beta\lambda \log H_t + \beta \Pi_{t+1} \\ &= \log c_t + (1-\lambda+\beta\lambda) \log H_t - \beta\lambda \log \frac{H_{t+1}}{\Gamma_{H,t+1}} + \beta \Pi_{t+1} \\ &= \log c_t + (1-\lambda+\beta\lambda) \log H_t - \beta\lambda \log \frac{1}{\Gamma_{H,t+1}} \\ &\quad + \beta \left(\Pi_{t+1} - \lambda H_{t+1}\right) \\ &= \log c_t + (1-\lambda+\beta\lambda) \log H_t + \beta\lambda \log \Gamma_{H,t+1} + \beta \left(\Pi_{t+1}'\right), \end{aligned}$$

in the last equation we still have the nonstationary term H_t, which would cause problems since H_{t+1} is contained in Π_{t+1}' and so on.

Figure A.1. Impulse Response to a Temporary Rise in Labor Productivity: Shutting down Endogenous Growth



However, if $1 - \lambda + \beta\lambda = 0$, then the H_t drops out (also out of future terms) and thus Π'_t becomes stationary. Thus, the condition for stationarity of Π'_t is $\lambda = 1/(1-\beta)$, and then the planner's goal to maximize Π_t is equivalent to maximizing

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log \Gamma_{H,t+1} + \beta \left(\Pi'_{t+1}\right) + \lambda \log H_t,$$

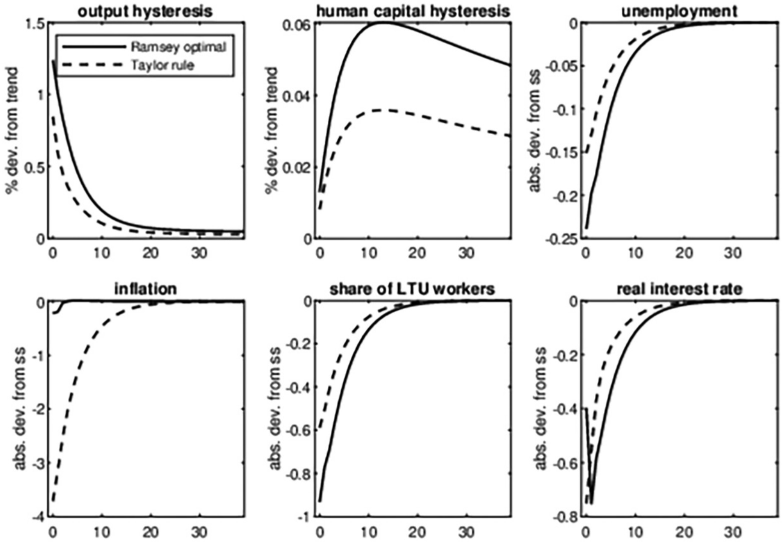
and since H_t is predetermined, i.e., out of the control of the planner at time t, this is equivalent to maximizing

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log \Gamma_{H,t+1} + \beta \left(\Pi'_{t+1}\right).$$

A.2 Limiting Cases

This appendix shows complementary figures (see Figures A.1 and A.2) that illustrate the full impulse responses for generating Figure 3.

Figure A.2. Impulse Response to a Temporary Rise in Labor Productivity: Shutting down Training Costs

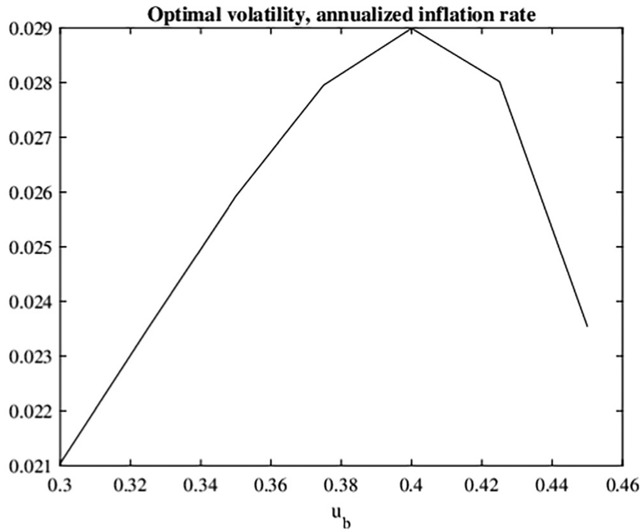


A.3 Optimal Inflation Volatility

Figure A.3 shows the sensitivity of optimal inflation volatility to the unemployment benefit parameter u_b. The lower bound for u_b is chosen for computational reasons—it is set such that the steady-state employment rate remains below unity.[15] The graph shows that optimal inflation volatility increases as unemployment benefits increase. In our model, unemployment benefits raise the wage above the efficient level and thus imply greater inefficiency in the labor market. This gives the Ramsey planner a stronger motive to use inflation actively to counteract this inefficiency over the business cycle.

[15] To avoid mixing different effects, we keep the rest of the calibration unchanged, which implies, however, that the unemployment rate changes.

Figure A.3. Unemployment Benefit and the Optimal Volatility of Inflation



References

Acemoglu, D., and R. Shimer. 1999. "Holdups and Efficiency with Search Frictions." *International Economic Review* 40 (4): 827–49.

Acharya, S., J. Bengui, K. Dogra, and S. L. Wee. 2022. "Slow Recoveries and Unemployment Traps: Monetary Policy in a Time of Hysteresis." *Economic Journal* 132 (646): 2007–47.

Annicchiarico, B., and L. Rossi. 2013. "Optimal Monetary Policy in a New Keynesian Model with Endogenous Growth." *Journal of Macroeconomics* 38 (B): 274–85.

Blanchard, O., and J. Galí. 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics* 2 (2): 1–30. http://ideas.repec.org/a/aea/aejmac/v2y2010i2p1-30.html.

Cerra, V., A. Fatás, and S. C. Saxena. 2023. "Hysteresis and Business Cycles." *Journal of Economic Literature* 61 (1): 181–225.

Chang, Y., J. Gomes, and F. Schorfheide. 2002. "Learning-by-Doing as a Propagation Mechanism." *American Economic Review* 92 (5): 1498–520.

Cheron, A. 2005. "Efficient v.s. Equilibrium Unemployment with Match-Specific Costs." *Economics Letters* 88 (2): 176–83.

Engler, P., and J. Tervala. 2018. "Hysteresis and Fiscal Policy." *Journal of Economic Dynamics and Control* 93 (August): 39–53.

Faia, E. 2009. "Ramsey Monetary Policy with Labour Market Frictions." *Journal of Monetary Economics* 56 (4): 570–81.

Faia, E., W. Lechthaler, and C. Merkl. 2014. "Labor Selection, Turnover Costs, and Optimal Monetary Policy." *Journal of Money, Credit and Banking* 46 (1): 115–44.

Gabaix, X. 2020. "A Behavioral New Keynesian Model." *American Economic Review* 110 (8): 2271–327.

Garga, V., and S. R. Singh. 2021. "Output Hysteresis and Optimal Monetary Policy." *Journal of Monetary Economics* 117 (January): 871–86.

Hagedorn, M., and I. Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98 (4): 1692–706.

Hall, R. E., and P. R. Milgrom. 2008. "The Limited Influence of Unemployment on the Wage Bargain." *American Economic Review* 98 (4): 1653–74.

Jones, L. E., R. E. Manuelli, and E. Stacchetti. 2000. "Technology (and Policy) Shocks in Models of Endogenous Growth." Staff Report No. 281, Federal Reserve Bank of Minneapolis.

Jordà, O., S. R. Singh, and A. M. Taylor. 2020. "The Long-Run Effects of Monetary Policy." NBER Working Paper No. 26666, January. https://ideas.repec.org/p/nbr/nberwo/26666.html.

Lechthaler, W., and D. Snower. 2013. "Quadratic Labor Adjustment Costs, Business Cycle Dynamics and Optimal Monetary Policy." *Macroeconomic Dynamics* 17 (2): 464–75.

Lechthaler, W., and M. Tesfaselassie. 2019. "A Note on Trend Growth, Unemployment and Optimal Monetary Policy." *Macroeconomic Dynamics* 23 (4): 1703–19.

———. 2023. "Endogenous Growth, Skill Obsolescence and Output Hysteresis in a New Keynesian Model with Unemployment." *Journal of Money, Credit, and Banking* 55 (8): 2187–213. https://doi.org/10.1111/jmcb.12979.

Miyamoto, H. 2011. "Efficiency in a Search and Matching Model with Training Costs." *Economic Modelling* 28 (4): 1838–41.

Pissarides, C. 2000. *Equilibrium Unemployment Theory*, 2nd ed. MIT Press.

———. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77 (5): 1339–69.

Ravenna, F., and C. Walsh. 2011. "Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear-Quadratic Framework." *American Economic Journal: Macroeconomics* 3 (2): 130–62.

Reifschneider, D., W. Wascher, and D. Wilcox. 2015. "Aggregate Supply in the United States: Recent Developments and Implications for the Conduct of Monetary Policy." *IMF Economic Review* 63 (1): 71–109.

Shimer, R. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (1): 25–49.

Stadler, G. W. 1990. "Business Cycle Models with Endogenous Technology." *American Economic Review* 80 (4): 763–78.

Thomas, C. 2008. "Search and Matching Frictions and Optimal Monetary Policy." *Journal of Monetary Economics* 55 (5): 936–56.

Walsh, C. 2003. "Labor Market Search and Monetary Shocks." In *Elements of Dynamic Macroeconomics*, ed. J. Altug, J. Chadha, and C. Nolan. Cambridge University Press.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.