# Disagreement and Discretionary Monetary Policy*

Marvin Goodfriend, Pierre Jinghong Liang, and Gaoqing Zhang
Carnegie Mellon University

This paper identifies a new coordination motive endogenously induced by a central bank's lack of commitment in the presence of information imperfection. We show that when differentially informed economic agents disagree about the central bank's inflation incentives, discretion in monetary policymaking induces agents to coordinate by "forecasting the forecasts of others" in order to forecast the central bank's policy actions. In particular, the induced coordination mechanism compels the central bank to choose monetary policy that responds to fluctuations in the average belief about its incentive. As a result, discretion has the potential to vastly increase fluctuations in employment and inflation, especially when the disagreement among agents is low. More broadly, our paper makes an argument for the inclusion of information diversity among agents in monetary policy discussions and in the characterization of the inflation dynamics.

JEL Codes: E52, E61, D82.

*Marvin Goodfriend passed away in December 2019. He was our colleague, mentor, and friend. We dedicate this paper to his memory and his contributions to the field of economics. Goodfriend was a faculty member of the Tepper School of Business at the Carnegie Mellon University and also had a research affiliation with NBER. Pierre Jinghong Liang and Gaoqing Zhang are from the Tepper School of Business at the Carnegie Mellon University. Pierre Liang gratefully acknowledges the Dean's Summer Research funding of the Tepper School. We thank Boragan Aruoba (editor), John McDermott, Pingyang Gao, Ariel Zetlin-Jones, Xiaodong Zhu, participants at Chicago-Minnesota Theory Conference and HKU Economics seminar, and especially two anonymous IJCB referees, for valuable comments.
Author e-mails: liangj@andrew.cmu.edu or gaoqingz@andrew.cmu.edu.

## 1. Introduction

This paper identifies a new coordination motive endogenously induced by a central bank's lack of commitment in the presence of information imperfection. We show that when differentially informed economic agents disagree about the central bank's inflation incentive, discretion in monetary policymaking induces agents to coordinate by "forecasting the forecasts of others" in order to forecast the central bank's policy actions. We abstract from inherent interdependencies that have been studied in the past to isolate the cause and the effect of the newly identified coordination motive.[1] In particular, our discretion-disagreement coordination mechanism compels the central bank to choose monetary policy that responds to fluctuations in the average belief about its inflation incentive which, in turn, is what forces agents to coordinate by forecasting the forecasts of others. As a result, discretion has the potential to vastly increase fluctuations in employment and inflation, especially when the disagreement among agents is low. More broadly, our paper makes an argument for the inclusion of information diversity among agents in monetary policy discussions and in the characterization of the inflation dynamics.

We adopt two information imperfections involving the central bank's inflation incentive: (i) disagreement among individual agents, and (ii) the average forecast error of all agents. Specifically, agents forecast future inflation incentives imperfectly and asymmetrically with a private signal that contains a common noise and an idiosyncratic noise. The common noise yields a stochastic average forecast error, and the volatility of the idiosyncratic noise governs dispersion of the individual forecasts around the average forecast. Surprisingly, we find that in equilibrium under discretion holding fixed average forecast accuracy, *more agreement* among agents destabilizes employment and inflation. Equally surprising, we find that *more accurate* average forecasts also destabilize employment and

[1] Coordination motives may also arise from inherent interdependencies among actors, either through technology linkage (Angeletos and Pavan 2004, 2007), information extraction (Townsend 1983), monopolistic competition (Woodford 2001), trading (Angeletos and La'O 2013), or beauty-contest preference (Morris and Shin 2002).

inflation in equilibrium under discretion when agents are sufficiently in agreement with each other. In effect, more agreement among agents coordinates forecasts more tightly and magnifies the effect of the common noise on employment and inflation via the central bank's more aggressive reaction to the average forecast. The magnified common noise constitutes an information-based source of macroeconomic instability which has not been identified previously.

The coordination problem we identify exists only under discretionary monetary policy. The problem goes away when the central bank follows a credible rule, even in the presence of imperfect information. Specifically, with commitment, the central bank can unilaterally and uniformly anchor each individual firm's expectation by credibly specifying both current and future policies. As a result, the central bank's equilibrium policy actions would be based upon the predetermined decision rule known to all firms. Although this rule may depend on future shocks to the inflation incentive that are imperfectly known to firms, firms can simply estimate these shocks by constructing first-order beliefs, without necessarily constructing (higher-order) beliefs about others' beliefs.

Without commitment, the central bank's current action loses control of the average current expectation of its future policy actions and, worse, it must react to its assessment of the average expectation. Therefore, when an individual agent forecasts future policy actions, the agent must forecast the average expectation which depends on other agents' forecasts. As a result, the average (first-order) belief of future policy actions now depends on the average forecast of other agents' forecasts, or the average second-order belief. Similarly, the average second-order belief would, in turn, depend on third-order beliefs, and so on.

In equilibrium under discretion, the aggregate variables such as output and inflation are functions of the average forecast of future monetary policy actions; the average forecast is determined by a hierarchy of higher-order beliefs which, in turn, depends on the properties of the average forecast and the degree of disagreement. Recognizing the complexity of the problem, we characterize the effects of higher-order beliefs on aggregate variables in a linear manner within a New Keynesian (New Synthesis) macroeconomic model, solved in closed form with a class of normally distributed signals.

With this specification, equilibrium inflation and aggregate output respond to the average forecast error linearly. The degree of disagreement, among other model parameters, affects the equilibrium sensitivity of inflation and output to the average forecast error because the degree of disagreement affects the aggressiveness with which the central bank, under discretion, is forced to respond to the average forecast error.

Given the equilibrium monetary policy, inflation and output become more volatile due to the addition of the shock on the average forecast. The induced coordination—the cause of heightened macrofluctuations—makes the problem especially pronounced due to a "multiplier" effect of the average forecast error. This is because the same private signal is used for each level (ladder) in the individual higher-order belief hierarchy; averaging across all individuals eliminates the idiosyncratic noise but not the common noise. Thus, the common noise is retained at each higher-order level of the average belief, magnifying the noise contained in the average forecast in equilibrium. When a discretionary central bank reacts to the average forecast, the magnified average forecast error enters into aggregate inflation and output, generating volatility due to the information imperfection beyond those "real" shocks commonly studied such as cost-push or demand shocks.

Facing such a pronounced problem caused by information imperfection, the conventional wisdom would suggest that reducing the volatilities of the information shocks would be desirable. We find that this intuition does not hold generally. Holding fixed the average forecast-error volatility, a higher degree of disagreement among agents makes them less coordinated, as each firm relies less on its signal when forming expectations about future policy actions. The central bank, in turn, becomes less responsive to the average forecast. Thus, fluctuations in employment and inflation due to information shocks are lower with a higher degree of disagreement. So narrowing the degree of disagreement would introduce more economic fluctuations and destabilize inflation and output.

On the other hand, holding the degree of disagreement fixed, an increase in the precision of the average forecast creates a trade-off between a direct reduction in the size of common noise and an indirect increase in sensitivity to the noise. Specifically, an increase in the precision directly reduces the common noise in the central bank's equilibrium monetary policy, leading to less volatility in the output and inflation series holding fixed the central bank's reaction sensitivity. However, when the precision of average forecast increases, all agents are better informed about future inflation and adjust their inflation expectation; the discretionary central bank's reaction to the aggregate expected inflation, which includes the error therein, adds more volatility to equilibrium inflation and output. When the degree of disagreement is high (low), the trade-off favors the direct (indirect) effect. Consequently, reducing the average forecast-error volatility stabilizes employment and inflation in equilibrium only when the degree of disagreement is high enough.

Our model specification borrows key elements from two distinct literatures: (i) macroeconomic research focusing on monetary policy and (ii) information economics research focusing on information structure. Based on the large literature on monetary policy research, we deploy the structural equations summarizing key insights from the New Keynesian model as described in the survey paper by Clarida, Gali, and Gertler (1999). Our model shares with Cukierman and Meltzer (1986), an early work on central bank opacity, a key feature that the central bank inflation incentive is stochastic and perpetually obscured. From the information economic research, we draw from research on information structure by economists as well as accounting researchers. The key element of our information structure—correlated private signals—has been used in the studies of financial markets (Holthausen and Verrecchia 1990) and recently has been studied in coordination settings with inherent interdependencies (Myatt and Wallace 2012; Liang and Zhang 2019).

To appreciate the connection this paper makes, consider the two debates in monetary policy that have received much academic, practical, and policy attention. The rules-versus-discretion debate has a long and varied standing. According to McCallum (1999, p. 1485), a "major reorientation" dates back to Barro and Gordon (1983a, 1983b) "built upon the insights of Kydland and Prescott (1977)." As is well known, the main insight identified by this literature is that discretionary policies suffer from the time-inconsistency problem: the market participants' rational expectation renders discretionary policies, designed by a benevolent central bank, ineffective and, worse, generates unnecessary inflation and economy fluctuations. The transparency–opacity debate in monetary policy can be traced back to Cukierman and Meltzer (1986) and Goodfriend (1986). This debate explicitly considers the potential information asymmetry between the central bank and the market participants (see, e.g., the survey by Geraats 2002). For example, collectively the public may perceive a lack of access into the workings of the central bank in promulgating monetary policy, leading to a perceived opacity of the central bank (e.g., Winkler 2002). Our paper bridges the two debates by identifying the link in between. In this light, our paper is related to that of Morris and Shin (2005), who also point to the connection between central bank discretion and transparency.[2] Interestingly, Morris and Shin (2005) also stress the preeminent role of managing expectations in linking the debate of central bank transparency and monetary policy to the extent that the central bank manipulates market expectations via communication and, at the same time, extracts information from market prices to guide monetary policy. In a sense, our paper complements the insight of Morris and Shin (2005) by outlining an alternative mechanism through which market expectations about the central bank's policy target interact with the monetary policy the central bank sets at its discretion.

Students of central banks have long noted the importance of the disagreement among individuals. For instance, Brunner (1981) studies the disagreement among individual agents' subjective perceptions of the monetary policy. King (1982, 1983) and Dotsey and King (1986) study the informational implication to monetary policy when differentially informed agents extract endogenous information from prices. Outside the two debates on central bank discretion and transparency, the pioneering idea by Phelps (1983) has stressed the lack of common knowledge in explaining the aggregate economic dynamics. The initial work by Townsend (1983) analytically formulated the idea of forecasting the forecasts of others. Woodford (2001), among other recent works, relies on finite information-processing capacity

[2] Morris and Shin (2005, p. 1) articulate a general point about this link from a political economy perspective: "In light of the considerable discretion enjoyed by independent central banks, the standards of accountability that they must meet are perhaps even higher than for most other public institutions. Transparency allows for democratic scrutiny of the central bank and hence is an important precondition for central bank accountability."

(Sims 2003) to show that informational disagreement among individuals leads them to construct beliefs about others' beliefs, or higher-order beliefs, within the contemporary framework of macromodels.

More recently, Benhabib, Wang, and Wen (2015) study an information friction caused by a timing friction: firms must make a production decision before demand is realized, while consumers must make labor supply and consumption plans before production is realized. This timing friction, coupled with aggregate sentiment in consumer demand, gives rise to endogenous aggregate fluctuations without any assumptions on technological externalities, non-convexities, etc. Absent in Benhabib, Wang, and Wen (2015) is the role of a central bank; thus the information friction we study is not the focus of their paper. The monetary policy is present in Paciello and Wiederholt (2014) in which information friction comes from the (in)attention to aggregate conditions paid by the firms, either exogenously or endogenously, when making individual production decisions in an economy described by standard New Keyesian model, the same framework we used. However, Paciello and Wiederholt (2014) consider a monetary policymaker with full commitment capabilities, an assumption we do not make in our paper. In Hellwig and Veldkamp (2009), the focus is on how exogenous coordination incentives affect agents' individual endogenous information acquisition, leading to the idea of "knowing what others know." The agents in our paper do indeed desire to know what others know (i.e., forecast of others' forecasts), but their coordination incentives are induced by a central bank unable to commit, as opposed to being exogenously given in Hellwig and Veldkamp (2009). In addition, the roles of diverse information and higher-order beliefs in large economies are also studied by Angeletos and Lian (2018), Angeletos and La'O (2020), and Angeletos and Huo (2021) in an emerging literature on incomplete information in macroeconomics (see Section 8 of the excellent survey by Angeletos and Lian 2016). However, they focus on dispersed information about the state of the economy as opposed to our focus on the dispersed information about the discretionary policy target of the central bank. Our paper complements this literature by adding a new coordination motive driven by central bank discretion and transparency.

The rest of the paper proceeds as follows. Section 2 lays out the basic macroeconomic framework and the key elements of our information assumptions. Section 3 analyzes the resulting model and constructs the central higher-order-belief arguments. Section 4 analyzes a parameterized version of the model. Section 5 concludes.

## 2. Model Setup

### *2.1 A Simple Macroeconomic Framework*

The economy is populated with a central bank that takes the nominal interest rate as the instrument of monetary policy, a representative household, and a continuum of firms, indexed by $[0, 1]$. Rather than deriving the optimal conditions for the household and firms, we describe the operation of the economy by a set of structural equations that can be derived from log-linearizing optimal consuming and profit-maximizing conditions (as in Galí 2008). Let $y_t$ and $y_t^p$ denote the logs of the aggregate economy output and the potential level of the output. The potential output is the level of output that would arise if wages and prices were fully flexible, but it may be lower than the efficient level due to existing frictions such as monopolistic competition, taxes, and subsidies. Define the output gap $x_t$ as the difference between $y_t$ and $y_t^p$:

$$x_t \equiv y_t - y_t^p. \tag{1}$$

In addition, let $\pi_t$ be the inflation rate from period $t-1$ to $t$.

First, there is a New Keynesian Phillips curve that links inflation $\pi_t$ to output gap $x_t$, generated by the firms in the economy:

$$\pi_t = \lambda x_t + \beta \bar{E}_t^F \pi_{t+1} + u_t, \tag{2}$$

where $\beta \in (0, 1)$ denotes the discounting factor and $\bar{E}_t^F [\cdot]$ denotes the average belief of the firms, i.e., $\bar{E}_t^F [\cdot] = \int_0^1 E_t \left[\cdot | I_t^i\right] di$, with firm $i$'s information set $I_t^i$.[3] The shock $u_t$ follows

$$u_t = \rho_u u_{t-1} + \hat{u}_t, \tag{3}$$

[3]For simplicity, we assume in our main analysis a reduced form of Phillips curve (2) in line with the standard New Keynesian Phillips curve when information is complete. As noted in Angeletos and Lian (2018) and Angeletos and Huo (2021), with incomplete information, the Phillips curve varies from the standard one. Accordingly, to assess the robustness of our analysis, we analyze in Section 5 a variant of our model with a microfounded Phillips curve.

where $\rho_u \in [0, 1)$ and $\hat{u}_t$ are i.i.d. random variables with mean 0 and variances $\sigma_u^2$. The Phillips curve can be derived by profit-maximizing conditions by firms that compete with each other monopolistically and face nominal price rigidities (Calvo 1983; Yun 1996; Woodford 2008). The key feature of the Phillips curve is that the average expected inflation $\bar{E}_t^F[\cdot]$ enters, which creates a role for the beliefs of the firms in affecting equilibrium inflation and output levels. This role, in turn, influences the central bank's monetary policy in equilibrium, making it partially self-fulling.

Second, there is a dynamic "IS" equation that describes the relation between real interest rate and output gap generated by the representative household in the economy:

$$x_t = -\phi r_t + E_t^H x_{t+1} + g_t, \tag{4}$$

where $r_t$ is the real interest rate (from period $t$ to period $t+1$) and $E_t^H[\cdot]$ denotes the expectation by the representative household. $g_t$ is a shock that follows,

$$g_t = \rho_g g_{t-1} + \hat{g}_t, \tag{5}$$

where $\rho_g \in [0, 1)$ and $\hat{g}_t$ are i.i.d. random variables with zero mean and variances $\sigma_g^2$. The IS equation can be derived from log-linearizing the Euler equation of the representative household.

Third, a Fisher equation links the nominal interest rate to the real interest rate and the representative household's expected inflation. Let $i_t$ be the nominal interest rate from period $t$ to $t+1$:

$$i_t = r_t + E_t^H \pi_{t+1}. \tag{6}$$

Replacing $r_t$ in the IS equation with $r_t = i_t - E_t \pi_{t+1}$ in the Fisher equation gives a modified IS equation,

$$x_t = -\phi \left(i_t - E_t^H \pi_{t+1}\right) + E_t^H x_{t+1} + g_t. \tag{7}$$

The central bank in period $t$ minimizes deviations of aggregate output gap and inflation from their respective targets:

$$\frac{1}{2} E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left[ \alpha \left(x_{t+\tau} - k_{t+\tau}\right)^2 + \pi_{t+\tau}^2 \right] \right\}, \tag{8}$$

subject to the Phillips curve (2) and the IS curve (7), where $\alpha$ is the relative weight on output deviations. We interpret the loss function as endowing the central bank a dual mandate: a zero-inflation target and output gap target. We assume that the target for the output gap is $k_t$. Adapting Barro and Gordon (1983a), $k_t$ represents the extent the central bank intends to raise actual output above potential (toward efficient output). For example, $k_t = 0$ implies that the central bank is satisfied with aggregate output at the potential output level (but below the efficient level).[4] When $k_t > 0$, the central bank has an incentive to target actual output above the potential level, generating an incentive to inflate. As is typically done, we will use $k_t$ to represent both higher output target than potential and inflation incentives interchangeably.

Critically, the inflation incentive is thus time varying in this paper, unlike in standard models such as that of Clarida, Gali, and Gertler (1999). In essence, this assumption implies that the central bank knows something more about its own preferred output gap target than the market collectively. Empirically, the assumption is consistent with the facts that financial markets respond to U.S. Federal Reserve (Fed) actions and market participants spend significant effort in Fed watching, a point made by a recent paper by Stein and Sunderam (2018). Further, there is a long literature on central bank secrecy/transparency dating back to the 1981 Supreme Court case favoring the Federal Open Market Committee's (FOMC's) position of delaying the release of meeting minutes, a practice criticized by academics such as Goodfriend (1986). While Fed transparency and accountability has increased after many years, some ambiguity and flexibility (about its own internal policy targets) remain in the process of Fed policymaking according to long-time observer Lars Svensson as recently as 2022 (see King and Wolman 2022). We believe these observations and past academic work support our assumption on the time-varying inflation incentive $k_t$.

[4]One may interpret a zero or low $k_t$ as either the central bank truly believes that potential output is very close to efficient output or that potential output is far below efficient output but a discretionary central bank recognizes its own limitation due to the time-inconsistence problem and chooses to tolerate the inefficiencies and thus lower inflation incentive.

### *2.2 Information Environment*

We first describe in detail the information environment, followed by a description of the resulting updating mechanism used by each firm when forecasting central bank's future inflation incentive. Every period, two standard macroshocks $\{u_t, g_t\}$ and the inflation incentive shock $k_t$ are contemporaneously observable to all firms, the representative household, and the central bank. The incentive shock $k_t$ follows,

$$k_t - \bar{k} = \rho_k \left(k_{t-1} - \bar{k}\right) + \nu_t, \tag{9}$$

where $\rho_k \in [0, 1)$ and $\nu_t \sim N(0, \frac{1}{q})$. As a result, $k_t \sim N\left(\bar{k}, \frac{1}{q_k}\right)$, where $\bar{k} > 0$ and $q_k = q\left(1 - \rho_k^2\right)$.

In addition, each individual firm receives a *private* foreknowledge about future inflation incentive. Specifically, at time $t$ firm $i$ receives a signal $s^i_{t+j}$ informative about the $j$-period-ahead inflation incentive shock $k_{t+j}$. The signal is modeled as

$$s^i_{t+j} = k_{t+j} + \eta_{t+j} + \varepsilon^i_{t+j}, \tag{10}$$

where $\eta_{t+j} \sim N\left(0, \frac{1}{m}\right)$ is common across firms and $\varepsilon^i_{t+j} \sim N\left(0, \frac{1}{n}\right)$ is idiosyncratic among firms.[5] Each private signal contains two shocks representing the two information imperfections. First, the average signal is a forecast of the future inflation incentive but with error, measured by $\eta_{t+j}$. Denoting $\bar{s}_{t+j} = \int_0^1 s^i_{t+j} di$ the average signal of all firms, we have

$$\text{Average Forecast Error: } \bar{s}_{t+j} - k_{t+j} = \eta_{t+j} \quad \text{and} \quad Var(\eta_{t+j}) = \frac{1}{m}. \tag{11}$$

The volatility of average forecast error is measured by its variance $\frac{1}{m}$. The larger $m$ is, the more precise $\bar{s}_t$ is about the central bank's

[5] $s^i_{t+j}$ can be interpreted as a sufficient signal summarizing any new information regarding $k_{t+j}$ that arrives in period $t$. It can be interpreted as from (unmodeled) private information acquisition, central bank disclosure, or learning from observing noisy signals of past inflation and output gap, etc. (Cukierman and Meltzer 1986; Stein and Sunderam 2018). Section 5 includes an extension that firms learn $k_{t+j}$ based on their observation of an endogenous aggregate variable.

incentive $k_t$. Second, the idiosyncratic shock in each signal generates disagreement among agents:

$$\text{Disagreement:} \quad s^i_{t+j} - \bar{s}_{t+j} = \varepsilon^i_{t+j} \qquad \text{and} \quad Var(\varepsilon^i_{t+j}) = \frac{1}{n}, \quad (12)$$

at any time $t$ and firm $i$. The degree of disagreement among firms is measured by $\frac{1}{n}$, the variance of $\varepsilon^i_t$. The larger $n$ is, the smaller the disagreement across firms. Notice that our specification of the information structure allows us to capture the precision of average forecast error independently from the disagreement among firms. Adopting an information structure capturing disagreement independently from average forecast error is critical for our model. If each private signal only contains idiosyncratic noise, the average forecast would be perfect by assumption, no matter what other imperfect public information is available.[6] In this regard, our modeling choice is motivated by insights generated by the decades of theoretical research on accounting information structure.[7]

[6]In effect, making this seemingly common and innocuous information assumption would inadvertently build in a collective rationality that precludes analysis of the kind of coordination mechanism that we study here. To see this more explicitly, consider an alternative two-signal structure as in Morris and Shin (2002). To fix ideas, suppose that each agent observes a purely public signal $z_{t+j} = k_{t+j} + \phi_{t+j}$ and a purely private signal $s^i_{t+j} = k_{t+j} + \chi^i_{t+j}$, where the noise terms are all independent of each other. Note that in this structure, we no longer have separate parameters capturing the degrees of disagreement and collective knowledge. To elaborate, note first that in the two-signal structure, collective knowledge is perfect through aggregating all the private signals $s^i_{t+j}$, i.e., the average of all private signals $\bar{s}_{t+j} = \int s^i_{t+j} di = k_{t+j}$. Second, the disagreement is jointly determined by the precision of the public and the private signals. To see this, note that each agent's posterior belief about $k_{t+j}$ is a weighted average of the public and the private signals. Intuitively, when the public signal becomes more precise, each agent places more weight on the public signal, resulting in less disagreement in their posteriors about $k_{t+j}$. Conversely, when the private signal becomes more precise, each agent places more weight on the private signal, contributing to more disagreement in their posteriors about $k_{t+j}$. Accordingly, given the different informational properties of the two-signal structure from those of our information structure, switching to the two-signal structure would alter the implications of our analysis for the roles of disagreement and collective knowledge considerably. We thank an anonymous reviewer for suggesting that we discuss the two-signal structure.

[7]Starting in the late 1960s and early 1970s, accounting researchers began linking accounting concepts to information economics concepts (see American

Every period, firm $i$ uses information set $I_t^i$ to forecast relevant future shocks in order to form beliefs about future inflation. We assume firm $i$'s relevant information set is

$$I_t^i = \left\{ \{u_\tau\}_{\tau=0}^{t}, \{g_\tau\}_{\tau=0}^{t}, \{k_\tau\}_{\tau=0}^{t}, \{s_\tau^i\}_{\tau=0}^{t+j} \right\}, \tag{13}$$

which includes all the past observations of $k_\tau$ up to period $t$ and all the past acquired signals $s_\tau^i$ up to period $t+j$.[8] Using $I_t^i$ to update beliefs about future $k$ follows Bayes's rule.[9]

Every period $t$, the central bank's information set is $I_t^{CB} = \{\{u_\tau\}_{\tau=0}^{t}, \{g_\tau\}_{\tau=0}^{t}, \{k_\tau\}_{\tau=0}^{t}, \{\bar{s}_\tau\}_{\tau=0}^{t+j}\}$, and it chooses policy instrument $i_t$ to achieve its objective.[10] This assumption is supported by the observation that a central bank is typically endowed with

---

Accounting Association monographs by Feltham 1973 and Mock 1976). The agenda is to build on the traditional approach under a purely measurement perspective and to tie the accounting measurement concepts to economic trade-off in decisionmaking under uncertainty. A seminal contribution is by Ijiri and Jaedicke (1966), who framed objectivity within statistical sampling setting as interpersonal agreement and related it to reliability. Ijiri and Jaedicke introduced two properties of accounting measurement structure. One is the distance between the true state and the average measurements, which we define as average forecast error in our paper. The other one is the distance between the average measurements and measurements by different measurers, which we define as disagreement.

[8]The sources of information available for each firm are exogenously given. We view $I_t^i$ as sufficient statistics for firm $i$ to forecast future inflation at time $t$. Endogenous information sources may include potentially noisy observations of output and prices such as nominal interest rate and inflation series. We abstract away from these endogenous sources to focus on the role of disagreement, however it is generated, on macrovariables.

[9]The computations of first-, second-, or higher-order expectations can be very simple or quite complex depending on parameters. Consider a simple case of $j = 2$ and $\rho_k = 0$; in order to form a first-order belief about next period's inflation incentive $k_{t+1}$, the firm $i$ would only use $s_{t+1}^i$ to compute its individual conditional expectation of $k_{t+1}$ (as all other signals are useless due to the independence assumptions). For a computation of the (higher-order) beliefs, see the discussion of Proposition 2 in Section 4. When $\rho_k$ is not zero, these expectation computations involve more terms, as more signals are now informative about future central bank incentives. For example, with a non-zero $\rho_k$, $\{k_t, s_{t+1}^i, s_{t+2}^i, \ldots s_{t+j}^i\}$ are all informative about $k_{t+1}$. See Proposition 2 in Section 4 for a detailed account of such first-, second-, and higher-order expectations when the inflation incentives $k_t$'s are serially correlated.

[10]Technically in a simultaneous-move game, a Nash equilibrium only requires the central bank to choose a best response to average expectations, not necessarily to observe the actual average expectation. Therefore, the observability of average signals by the central bank is inconsequential.

more information than an individual firm. The representative household's information set is $I_t^H = \left\{ \{u_\tau\}_{\tau=0}^t , \{g_\tau\}_{\tau=0}^t , \{k_\tau\}_{\tau=0}^t \right\}$, and it chooses intertemporal consumption with rational expectation. As we will show later, the Phillips curve that effectively determines the equilibrium inflation and output gap does not include the expectation of the representative household $E_t^H[\cdot]$. As a result, $E_t^H[\cdot]$ (thus the household's information set) affects nominal interest rate through the dynamic IS curve but does not affect the equilibrium inflation and output.

## 3. Preliminary Policy Analysis with Disagreement and Discretion

We assume that the central bank conducts a discretionary monetary policy each period. In a typical period $t$, the firms and the central bank simultaneously decide their actions. Specifically, the central bank chooses the nominal interest $i_t$ given its information set $I_t^{CB}$, while each firm forms an expectation (forecasts) about the inflation rate in the next period, given its information set $I_t^i$ and its conjecture about the central bank's future actions. In short, the players play a simultaneous-move game according to their best response given their own information set. This section provides the preliminary analysis needed to construct the closed-form equilibrium outcome in Section 4.

### *3.1 First-Order Condition for the Central Bank*

Since the central bank cannot commit, it only chooses the current nominal interest rate $i_t$ (but not future rates) that solves the following optimization program:

$$\begin{aligned} \min_{i_t} \quad & \frac{1}{2} E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left[ \alpha \left( x_{t+\tau} - k_{t+\tau} \right)^2 + \pi_{t+\tau}^2 \right] \right\}, \\ s.t. \quad & x_t = -\phi \left[ i_t - E_t^H \pi_{t+1} \right] + E_t^H x_{t+1} + g_t, \\ & \pi_t = \lambda x_t + \beta \bar{E}_t^F \pi_{t+1} + u_t. \end{aligned} \tag{14}$$

Following Clarida, Gali, and Gertler (1999), we solve the optimization program in two stages: first, we solve for the pair of $(x_t, \pi_t)$ that maximizes the objective given the Phillips curve (2); second, we use the IS curve (7) to determine the nominal interest rate $i_t$ that supports the optimal pair of $(x_t, \pi_t)$. Throughout the paper, since we are mostly interested in the equilibrium properties of inflation and output, we will focus on analyzing the first stage. Accordingly, we will omit the superscript $F$ in expectation notation $\bar{E}_t^F$ and use $\bar{E}_t$ to represent average expectation by the firms in the rest of the paper to simplify exposition. In the first stage, notice that since the central bank, under discretion, cannot credibly change the firms' beliefs about its future actions, it takes the firms' expectations as given. As a result, the optimization problem for the central bank can be simplified into

$$\begin{aligned} &\min_{\{x_t,\pi_t\}} \frac{1}{2}\left[\alpha\left(x_t-k_t\right)^2+\pi_t^2\right]+F_t, \\ s.t. \quad &\pi_t=\lambda x_t+f_t, \end{aligned} \tag{15}$$

where $f_t = \beta\bar{E}_t\pi_{t+1} + u_t$, and $F_t = \frac{1}{2}E_t\left\{\sum_{\tau=1}^{\infty}\beta^i\left[\alpha\left(x_{t+\tau}-k_{t+\tau}\right)^2 + \pi_{t+\tau}^2\right]\right\}$.[11] The first-order condition on $x_t$ gives

$$x_t = -\frac{\lambda}{\alpha}\pi_t + k_t. \tag{16}$$

The first-stage solution reveals that the central bank must choose its policy instrument (in the second stage) to respect Equation (16). Holding $k_t$ constant, a central bank seeing a positive (cost-push) shock $u_t$ that pushes current inflation $\pi_t$ higher via the Phillips curve would choose a policy to reduce current output, thus lowering the output gap $x_t$. Equation (16) also shows that the central bank is tempted to raise the output gap by $k_t$, holding the (cost-push) shock $u_t$ constant. The higher the $k_t$, the higher the central bank's temptation to push up the output gap.

[11] To focus our attention on the role of higher-order beliefs on monetary policy, we ignore alternative equilibria involving reputation (using, e.g., grim-trigger strategies) which could support a more efficient outcome (see text by Mailath and Samuelson 2006). See footnote 26 on page 1671 of Clarida, Gali, and Gertler (1999) for background and explanations.

Substituting the first-order condition (16) into the Phillips curve (2) reveals inflation expectation dynamics generated by the central bank's best response:

$$\pi_t = \frac{\alpha\lambda}{\alpha+\lambda^2}k_t + \frac{\alpha\beta}{\alpha+\lambda^2}\bar{E}_t\left[\pi_{t+1}\right] + \frac{\alpha}{\alpha+\lambda^2}u_t. \tag{17}$$

Equation (17) suggests that the central bank must respond to changes in average expectations $\bar{E}_t\left[\pi_{t+1}\right]$ in determining inflation. The higher the expected future inflation $\bar{E}_t\left[\pi_{t+1}\right]$, the higher the actual current inflation $\pi_t$. In this sense, (17) captures the self-fulfilling nature in monetary policymaking. The coefficient before $\bar{E}_t\left[\pi_{t+1}\right]$, $\frac{\alpha\beta}{\alpha+\lambda^2} \in (0,1)$, thus measures how responsive the actual current inflation is to the expected future inflation.

### *3.2 Forward-Recursive Solutions of Phillips Curve under Disagreement and Discretion*

We solve for $\pi_t$ through forward-looking iteration. Iterating (17) once gives

$$\begin{aligned}\pi_t = {} & \frac{\alpha}{\alpha+\lambda^2}u_t + \frac{\alpha\beta}{\alpha+\lambda^2}\frac{\alpha}{\alpha+\lambda^2}\bar{E}_t\left[u_{t+1}\right] + \frac{\alpha\lambda}{\alpha+\lambda^2}k_t \\ & + \frac{\alpha\beta}{\alpha+\lambda^2}\frac{\alpha\lambda}{\alpha+\lambda^2}\bar{E}_t\left[k_{t+1}\right] + \left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^2\bar{E}_t\bar{E}_{t+1}\left[\pi_{t+2}\right]. \end{aligned} \tag{18}$$

The key observation is that, in contrast to symmetric information case (i.e., no disagreement), the law of iterated expectation does not hold for average beliefs by differentially informed firms (Morris and Shin 2002).[12] That is,

$$\bar{E}_t\bar{E}_{t+1}\left[\cdot\right] \neq \bar{E}_t\left[\cdot\right]. \tag{19}$$

In fact, $\bar{E}_t\bar{E}_{t+1}\left[\cdot\right]$ corresponds to the second-order average beliefs of the firms, i.e., the firms' beliefs about the others' beliefs, which may differ substantially from the first-order average beliefs $\bar{E}_t\left[\cdot\right]$ when

[12]We will verify this point once we specify information structure for firms in the next section.

firms are differentially informed about central bank incentives. Similarly, a third-order belief term would show up when Equation (17) is iterated twice, and so on. Because of the failure of the law of iterated expectation, we must characterize the entire hierarchy of higher-order beliefs, all of which depend on the firms' current information set ($I_t^i$) and affect the equilibrium monetary policies. To simplify notations, we denote the $l$-th order beliefs as $\bar{E}_t^l\left[\cdot\right]$, where

$$\bar{E}_t^l\left[\cdot\right] \equiv \bar{E}_t \bar{E}_{t+1} ... \bar{E}_{t+l-1}\left[\cdot\right]. \tag{20}$$

We find that the iteration of (17) converges and gives $\pi_t$ as a function of the higher-order beliefs, as summarized in the proposition below.

PROPOSITION 1. *In equilibrium, the inflation rate $\pi_t$ depends on the sum of the higher-order beliefs about $\{k_{t+l}\}_{l=0}^{l=\infty}$, i.e.,*

$$\begin{aligned}\pi_t = {} & \frac{\alpha\lambda}{\alpha+\lambda^2}k_t + \frac{\alpha u_t}{\alpha\left(1-\beta\rho_u\right)+\lambda^2} \\ & + \left\{\sum_{l=1}^{\infty}\left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^l \frac{\alpha\lambda}{\alpha+\lambda^2}\bar{E}_t^l\left[k_{t+l}\right]\right\}.\end{aligned} \tag{21}$$

Before we consider the specific linear-normal information structure laid out earlier, we note that the coordination problem we identify exists only under discretionary monetary policy. The problem goes away when the central bank follows a credible rule, even in the presence of imperfect information. Specifically, with commitment, the central bank can unilaterally and uniformly *anchor* each individual firm's expectation by credibly specifying both current and future policies $\{\pi_{t+\tau}\}_{\tau=0}^{\infty}$. As a result, the central bank's equilibrium policy actions would be based upon the predetermined decision rule that is known to all firms. In equilibrium, firms' forecasting problem would then reduce to simply estimating the unobservable shocks in the predetermined rule—for instance, the central bank's inflation incentives $\{k_{t+\tau}\}_{\tau=0}^{\infty}$—by constructing first-order beliefs about them, without necessarily constructing (higher-order) beliefs about others' beliefs.

Without commitment, however, the central bank's current action loses control of the average current expectation of its future policy actions and, worse, it must react to its assessment of the average expectation. All firms know that the central bank will adjust its actions in response to the firms' aggregate expectations, $\bar{E}_t[\pi_{t+1}]$, every period. In mechanical terms, the central bank will choose the pair of $\{x_t, \pi_t\}$ for a given $\bar{E}_t[\pi_{t+1}]$, the aggregate expectation of its own future action, based on the Phillips curve relation. From an individual firm's perspective, since others' forecasts collectively affect the central bank's monetary actions—which, in turn, affect the very inflation rate it wants to forecast to begin with—it must also forecast the forecasts of others. In this process, rationality dictates that it must form beliefs about others' beliefs about $\{k_{t+\tau}\}_{\tau=0}^{\infty}$, others' beliefs about others' beliefs, and even higher-order beliefs. These beliefs in turn determine individual forecasts of all firms, which collectively influence the equilibrium inflation through the self-fulfilling feature embedded in the modified Phillips curve (17). Notice that from Equation (21), the relative importance of higher-order beliefs is determined by $\frac{\alpha\beta}{\alpha+\lambda^2}$, the responsiveness of the actual inflation to the expected future inflation. If $\frac{\alpha\beta}{\alpha+\lambda^2} = 0$, the equilibrium inflation becomes independent of the aggregate expectation, making it unnecessary for each firm to forecast others' forecasts. As a result, all the higher-order-belief terms vanish.

### *3.3 A Closed-Form Forward-Recursive Solution*

As a matter of exposition and practice, we believe allowing two-period-ahead foreknowledge (i.e., setting $j = 2$) is sufficient, in part, because it allows a closed-form solution to the full equilibrium (the derivations for the cases with $j > 2$ are similar but less analytically tractable).[13] Specifically, at any period $t$, a firm's information set is $I_t^i = \left\{ \{u_\tau\}_{\tau=0}^{t}, \{g_\tau\}_{\tau=0}^{t}, \{k_\tau\}_{\tau=0}^{t}, \left\{s_\tau^i\right\}_{\tau=0}^{t+2} \right\}$. To proceed, we first remove redundant elements in the firm's information set. First, at

[13]As it turns out, if firms only have one-period-ahead foreknowledge about the inflation incentive (i.e., $j = 1$), the higher-order beliefs would become degenerate such that all higher-order beliefs would coincide with the first-order beliefs. Accordingly, absent non-degenerate higher-order beliefs, firms' signals would be used to forecast the inflation incentive but not the forecasts of other firms. Hence the implication of our model for central bank transparency would also change. Detailed analysis is available upon request.

each period $t$, observe that $\{k_\tau\}_{\tau=0}^{t}$ are commonly known and are sufficient statistics for signals, $\left\{s_\tau^i\right\}_{\tau=0}^{t}$, so the only useful signals are $\left\{s_{t+1}^i, s_{t+2}^i\right\}$ when forecasting future $k$'s. Second, since $k_t$ follows an AR(1) process, $k_t$ is a sufficient statistics for all the past $\{k_\tau\}_{\tau=0}^{t-1}$. To sum, a firm's information set can be simplified into $I_t^i = \left\{k_t, s_{t+1}^i, s_{t+2}^i\right\}$ for the purpose of forecasting future $k$'s.

The following proposition provides the closed-form solutions to the higher-order beliefs terms:

PROPOSITION 2. *When $j = 2$, the $l$-th order average beliefs become*

$$\bar{E}_t^l \left[k_{t+l}\right] = \bar{k} + \rho_k^{l-1} \left\{ \left[1 - w\left(l\right)\right] \bar{E}_t \left[k_{t+1} - \bar{k} | s_{t+1}^i, k_t\right] + w\left(l\right) \frac{\bar{s}_{t+2} - \bar{k}}{\rho_k} \right\}, \tag{22}$$

*where* $\bar{E}_t \left[k_{t+1} - \bar{k} | s_{t+1}^i, k_t\right] = \frac{q}{q + \frac{mn}{m+n}} \rho_k \left(k_t - \bar{k}\right) + \frac{\frac{mn}{m+n}}{q + \frac{mn}{m+n}} \left(\bar{s}_{t+1} - \bar{k}\right)$ *and $w\left(l\right)$ is a constant given in the appendix.*

Proposition 2 suggests that $\bar{E}_t \left[k_{t+1} | s_{t+1}^i, k_t\right]$ and $\bar{s}_{t+2}$ are the two sufficient statistics for period-$t$ firms to forecast the average higher-order beliefs about the central bank's future inflation incentive. To further illustrate the construction of the higher-order-belief hierarchy, consider first a special case in which the central bank's inflation incentive $k_t$ is serially uncorrelated ($\rho_k = 0$). In this case, firms share a common prior on $k_t \sim N\left(\bar{k}, \frac{1}{q}\right)$. In addition, when forecasting $k_{t+l}$, the only useful signals are the prior $\bar{k}$ and $s_{t+l}^i$, and all the other signals, $\left\{s_\tau^i\right\}_{\tau \neq t+l}^{t}$, are not useful, since $k_t$ is serially uncorrelated. In this case, the first-order belief $\bar{E}_t \left[k_{t+1}\right]$ is a weighted average of the prior and the average signal $\bar{s}_{t+1}$, with the weights simply the ones under Bayesian updating and similarly for the first-order belief $\bar{E}_t \left[k_{t+2}\right]$, i.e.,

$$\bar{E}_t \left[k_{t+1}\right] = \bar{k} + \frac{\frac{1}{q}}{\frac{1}{q} + \frac{1}{m} + \frac{1}{n}} \left(\bar{s}_{t+1} - \bar{k}\right), \tag{23}$$

$$\bar{E}_t \left[k_{t+2}\right] = \bar{k} + \frac{\frac{1}{q}}{\frac{1}{q} + \frac{1}{m} + \frac{1}{n}} \left(\bar{s}_{t+2} - \bar{k}\right). \tag{24}$$

To form the average second-order belief, $\bar{E}_t^2\left[k_{t+2}\right]$, first consider an individual firm $i$'s expectation of next period's average belief:

$$\begin{aligned} E_t^i\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right] &= \bar{k} + \frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(E_t^i\left[\bar{s}_{t+2}\right]-\bar{k}\right) \\ &= \bar{k} + \frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(\bar{k}+\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(s_{t+2}^i-\bar{k}\right)-\bar{k}\right) \\ &= \bar{k} + \frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(s_{t+2}^i-\bar{k}\right), \qquad (25) \end{aligned}$$

and aggregating all firms' expectations, the average second-order belief becomes[14]

$$\begin{aligned} \bar{E}_t^2\left[k_{t+2}\right] &\equiv \bar{E}_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right] \\ &= \bar{k} + \frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(\bar{s}_{t+2}-\bar{k}\right). \qquad (26) \end{aligned}$$

Notice that in forming the average second-order belief $\bar{E}_t^2\left[k_{t+2}\right]$, the average signal $\bar{s}_{t+2}$ is assigned a lower weight relative to the typical Bayesian weight, i.e., $\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}} < \frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}$, while the prior is assigned a higher weight. As a result, $\bar{E}_t\left[k_{t+2}\right] \neq \bar{E}_t^2\left[k_{t+2}\right]$, consistent with literature on the role of public information in coordination settings (Morris and Shin 2002). Notice in a standard model without disagreements ($n = \infty$), each firm's information set contains only public information; no "overweighting" takes place, making the higher-order-beliefs degenerate. In the special case of $\rho_k = 0$, for beliefs higher than the second order, the higher-order expectations become degenerate and equal to the prior $\bar{k}$, i.e., $\bar{E}_t^l\left[k_{t+l}\right] \equiv \bar{k}$ for $l > 2$. This is because period-$t$ firms only receive private signals about, and thereby disagree on, the central bank's future inflation incentive up to period $t+2$. For any other future $k_{t+l}$, period-$t$ firms share the same common prior $\bar{k}$ and agree with each other. Such

[14]One can also verify that taking a limit of the expression of the higher-order beliefs, i.e., expression (22), in Proposition 2 at $\rho_k = 0$ produces the same expressions of $\bar{E}_t\left[k_{t+1}\right]$ and $\bar{E}_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right]$ given in the text.

perfect agreement among firms makes the higher-order beliefs that are higher than the second-order degenerate.

In the general case of serially correlated inflation incentive $k_t$ ($\rho_k \neq 0$), the entire hierarchy of higher-order beliefs, including the ones that are higher than the second-order belief, remain non-degenerate. This is because, since $k_t$ is serially correlated, period-$t$ firms can utilize their private signals about $k_{t+1}$ and $k_{t+2}$ to forecast future $\{k_{t+l}\}_{l=3}^{\infty}$, thus disagreeing with each other in their beliefs about all of the central bank's future inflation incentive. Such disagreement in turn makes the higher-order beliefs $\left\{\bar{E}_t^l\left[k_{t+l}\right]\right\}_{l=3}^{\infty}$ non-degenerate. The proof of Proposition 2 contains the derivation of these expectations explicitly.

### *3.4 Symmetric Information Benchmark*

Before we characterize the equilibrium in the model with informational imperfection, for comparison purposes, consider an identical model except no firm receives any signal about future inflation incentives (see Clarida, Gali, and Gertler 1999, Sections 3 and 4.1, for similar results.). It is well known in this setting that under discretionary monetary policy, the equilibrium output and inflation contain an inflation bias driven by $k_t$ and $\bar{k}$:

$$\begin{aligned}
\pi_t^* &= \frac{\alpha}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}u_t + \frac{\alpha\lambda}{\alpha\left(1-\beta\right)+\lambda^2}\bar{k} \\
&\quad + \frac{\alpha\lambda}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\left(k_t-\bar{k}\right), \\
x_t^* &= -\frac{\lambda}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}u_t + \frac{\alpha\left(1-\beta\right)}{\alpha\left(1-\beta\right)+\lambda^2}\bar{k} \\
&\quad + \frac{\alpha\left(1-\beta\rho_k\right)}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\left(k_t-\bar{k}\right), \\
i_t^* &= \frac{g_t}{\phi} + \frac{\alpha\lambda}{\alpha\left(1-\beta\right)+\lambda^2}\bar{k} + \frac{\alpha\rho_u+\lambda\left(1-\rho_u\right)/\phi}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}u_t \\
&\quad + \frac{\alpha\lambda\rho_k-\alpha\left(1-\beta\rho_k\right)\left(1-\rho_k\right)/\phi}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\left(k_t-\bar{k}\right).
\end{aligned} \tag{27}$$

The sources of aggregate output and inflation fluctuations are shocks to the Phillips curve and the central bank's inflation incentive. Equilibrium nominal interest rate also reacts to shocks to the dynamic IS curve.

## 4. Equilibrium in Closed Form

In this section, we first derive, in closed form, the equilibrium inflation, output gap, and nominal interest rate under the imperfect information environment for the special case of $j = 2$ (i.e., firms receive only two-period-ahead $k_{t+2}$). Then, we conduct comparative stochastic dynamic analysis of how information imperfections affect the volatilities of equilibrium inflation and output.

### *4.1 The Stochastic Stationary Equilibrium*

Substituting the expressions for the higher-order expectations $\bar{E}_t^l\left[k_{t+l}\right]$ given by Equation (22) into the solution for $\pi_t$ (in Equation (21)) and then $x_t$ (in Equation (16)) gives the equilibrium inflation $\pi_t^{**}$ and output gap $x_t^{**}$. The equilibrium nominal interest rate can be derived by substituting the pair $(\pi_t^{**}, x_t^{**})$ into the IS curve (7), giving the complete equilibrium characterization.

PROPOSITION 3. *Assuming $j = 2$, the equilibrium $\{\pi_t^{**}, x_t^{**}, i_t^{**}\}$ is given by*

$$\begin{aligned}
\pi_t^{**} = {} & \frac{\alpha u_t}{\alpha\left(1-\beta\rho_u\right)+\lambda^2} + \frac{\alpha\lambda}{\alpha\left(1-\beta\right)+\lambda^2}\bar{k} \\
& + \frac{\alpha\lambda}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\left(k_t-\bar{k}\right) \\
& + \frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1} \\
& \times\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right),
\end{aligned}$$

$$
\begin{aligned}
x_t^{**} &= -\frac{\lambda u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\lambda^2}\bar{k} \\
&\quad + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right) \\
&\quad - \frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1} \\
&\qquad \times \left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right), \\
i_t^{**} &= \frac{g_t}{\phi} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\rho_u+\lambda(1-\rho_u)/\phi}{\alpha(1-\beta\rho_u)+\lambda^2}u_t \\
&\quad + \frac{\alpha\lambda\rho_k-\alpha(1-\beta\rho_k)(1-\rho_k)/\phi}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right) \\
&\quad + \frac{1}{\phi}\left[\frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\right. \\
&\qquad \left.\times \left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right)\right], \qquad (28)
\end{aligned}
$$

*where the "demeaned" higher-order beliefs are*

$$
\begin{aligned}
\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right) &= \left[1-w(l)\right]\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\nu_{t+1}+\eta_{t+1}\right) \\
&\quad + w(l)\left(\frac{\nu_{t+2}+\rho_k\nu_{t+1}+\eta_{t+2}}{\rho_k}\right). \qquad (29)
\end{aligned}
$$

Proposition 3 shows that the equilibrium inflation $\pi_t^{**}$ and hence the output gap $x_t^{**}$ are determined by three factors, the contemporaneous (cost-push) shock $u_t$, the inflation bias $\frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right)$, and the higher-order expectations $\bar{E}_t^l\left[k_{t+l}\right]$. The first two factors have been extensively examined in the literature and appear even in the benchmark model without informational imperfection (see Equation (27)). Specifically, consistent with standard results (Clarida, Gali, and Gertler 1999), we verify that the cost-push shock $u_t$ is inflationary. In addition, consistent with Barro and Gordon (1983a), we find that the discretion in monetary policy can lead to a persistent inflation bias $\frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k}$.

In addition to the two well-known effects in the literature, the proposition above shows that the combination of the discretion in the monetary policy and the disagreement among firms can lead to another potentially detrimental effect, as captured in the third terms of the equilibrium inflation and output gap. Through the channel of the induced-coordination problem we identify, the discretionary monetary policy causes the equilibrium inflation and output to react to firms' higher-order beliefs about the central bank's inflation incentive, leading to heightened fluctuations in output and inflation. Our findings thereby suggest that the combination of lack of commitment by the central bank and the imperfect information known to firms makes the central bank less capable to stabilize output and inflation, compared with a central bank in an alternate economy without such commitment and information frictions.

The source of the heightened output and inflation fluctuations comes from the volatilities of the primitive variables in our model. Specifically, the equilibrium inflation and output will respond not only to the central bank's current inflation incentive $k_t$ but also to the noises, $\{\eta_{t+1}, \eta_{t+2}\}$, contained in firms' average signals $\{\bar{s}_{t+1}, \bar{s}_{t+2}\}$, as well as innovations in the central bank's future inflation incentive, $\{\nu_{t+1}, \nu_{t+2}\}$. Furthermore, the coordination problem induced by the discretionary monetary policy makes the destabilizing effect of the monetary policy more prominent, due to a "multiplier" effect. When forecasting the forecasts of others, firms' average forecast is determined by a hierarchy of higher-order beliefs, each of which depends on the noises in firms' current information. As the monetary policy reacts to firms' forecast, the entire hierarchy of higher-order beliefs enters into the equilibrium inflation and output and the noise contained in these beliefs leads to heightened volatility. In particular, Equation (28) shows precisely that since the same private signals $\left\{s^i_{t+1}, s^i_{t+2}\right\}$ are used for each level in the individual higher-order belief hierarchy, the common information noise $\{\eta_{t+1}, \eta_{t+2}\}$ in these private signals is retained at every term of the higher-order beliefs, magnifying the noises contained in the average inflation forecast. When the central bank responds to the average forecast, the magnified information noises enter into aggregate inflation and output, generating heightened macrofluctuations.

## *4.2 Comparative Stochastic Dynamic Analysis*

In the face of the heightened volatility caused by firms' imperfect information, the conventional wisdom would suggest that reducing the volatilities of the two informational shocks, $\eta_{t+j}$ (average forecast error) and $\varepsilon_{t+j}^{i}$ (degree of disagreement), would be desirable. We find that this intuition does not hold generally. Importantly, we show that reducing the volatilities of the two informational shocks can increase the macrofluctuations by inducing a more aggressive monetary policy response. To see the effect of informational properties on volatilities, from Proposition 3, the volatility of inflation is computed as

$$Var\left(\pi_t^{**}\right) = \left(\frac{\alpha}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}\right)^2 \frac{\sigma_u^2}{1-\rho_u^2} + \left(\frac{\alpha\lambda}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\right)^2 Var\left(k_t\right) + \left(\frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\right)^2 \times Var\left(\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l[k_{t+l}]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right)\right), \tag{30}$$

where the first term represents the volatility stemming from the shock $u_t$, the second term represents the volatility stemming from the central bank's *current* inflation incentive $k_t$, and the third term represents the volatility stemming from higher-order expectations about the central bank's *future* inflation incentive. By the first-order condition, $x_t^{**} = -\frac{\lambda}{\alpha}\pi_t^{**} + k_t$, the volatility of $x_t^{**}$ is proportional to the volatility of $\pi_t^{**}$, and the two share similar properties. Thus we will focus on analyzing the volatility of $\pi_t^{**}$. For notational convenience, we define the sensitivities of the equilibrium inflation to the future signals $\bar{s}_{t+1}$ and $\bar{s}_{t+2}$ as

$$W_{\bar{s}_{t+1}}\left(m,n\right) = \frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1} \times \left\{\left[1-w\left(l\right)\right]\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\right\}, \tag{31}$$

$$W_{\bar{s}_{t+2}}\left(m,n\right)=\frac{\alpha\lambda}{\alpha+\lambda^{2}}\frac{\alpha\beta}{\alpha+\lambda^{2}}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_{k}}{\alpha+\lambda^{2}}\right)^{l-1}\frac{w\left(l\right)}{\rho_{k}}, \tag{32}$$

which depend on the informational properties, the average forecast error $m$, and the degree of disagreement $n$. Using notations $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$, $Var\left(\pi_t^{**}\right)$ can be rewritten as

$$\begin{aligned}
&\left(\frac{\alpha}{\alpha\left(1-\beta\rho_{u}\right)+\lambda^{2}}\right)^{2}\frac{\sigma_{u}^{2}}{1-\rho_{u}^{2}}+\left(\frac{\alpha\lambda}{\alpha\left(1-\beta\rho_{k}\right)+\lambda^{2}}\right)^{2}Var\left(k_{t}\right)\\
&+\left[W_{\bar{s}_{t+1}}\left(m,n\right)+\rho_{k}W_{\bar{s}_{t+2}}\left(m,n\right)\right]^{2}Var\left(\nu_{t+1}\right)\\
&+\left[W_{\bar{s}_{t+1}}\left(m,n\right)\right]^{2}Var\left(\eta_{t+1}\right)\\
&+\left[W_{s_{t+2}}\left(m,n\right)\right]^{2}\left[Var\left(\nu_{t+2}\right)+Var\left(\eta_{t+2}\right)\right].
\end{aligned} \tag{33}$$

$Var\left(\nu_{t+1}\right)=Var\left(\nu_{t+2}\right)=\frac{1}{q}$ and $Var\left(\eta_{t+1}\right)=Var\left(\eta_{t+2}\right)=\frac{1}{m}$. Therefore, $Var\left(\pi_t^{**}\right)$ becomes

$$\begin{aligned}
&\left(\frac{\alpha}{\alpha\left(1-\beta\rho_{u}\right)+\lambda^{2}}\right)^{2}\frac{\sigma_{u}^{2}}{1-\rho_{u}^{2}}+\left(\frac{\alpha\lambda}{\alpha\left(1-\beta\rho_{k}\right)+\lambda^{2}}\right)^{2}Var\left(k_{t}\right)\\
&\quad+\frac{\left[W_{\bar{s}_{t+1}}\left(m,n\right)+\rho_{k}W_{\bar{s}_{t+2}}\left(m,n\right)\right]^{2}+\left[W_{s_{t+2}}\left(m,n\right)\right]^{2}}{q}\\
&\quad+\frac{\left[W_{\bar{s}_{t+1}}\left(m,n\right)\right]^{2}+\left[W_{s_{t+2}}\left(m,n\right)\right]^{2}}{m}.
\end{aligned} \tag{34}$$

Equation (34) suggests that in addition to the volatility driven by the shocks $u_t$ and $k_t$, inflation volatility is also driven by two other shocks. The third term in (34) represents the fundamental volatility stemming from the innovations in the central bank's future inflation incentive $\{\nu_{t+1}, \nu_{t+2}\}$, and the fourth term is the non-fundamental volatility stemming from the noises in firms' signals, i.e., $\{\eta_{t+1}, \eta_{t+2}\}$. Equation (34) shows that the informational properties can influence the macrofluctuations in two ways. First, improving the precision of the average forecast error (increasing $m$)

directly reduces the size of the noises $\{\eta_{t+1}, \eta_{t+2}\}$ in the equilibrium monetary policy, leading to less volatility in the output and inflation. We capture this effect in the $\frac{1}{m}$ term in (34) and call this effect a *noise-diminishing* effect. Second, when either the precision of average forecast error or the agreement among firms increases, the average forecast becomes more sensitive to the firms' imperfect information, thus the average forecast error. The central bank, in turn, reacts more aggressively to the average expectation; unfortunately, this reaction adds more volatility to equilibrium inflation and output. In other words, increasing $m$ or $n$ can increase the sensitivity of the monetary policy to firms' signals and noises (i.e., $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$). We capture this effect in $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$ and call it a *sensitivity* effect. Whether improving firms' information (increasing $m$ and $n$) reduces the volatilities thereby depends on the trade-off between the sensitivity effect and the noise-diminishing effect. We summarize the effect of the informational properties on the volatilities of inflation and output in the proposition below.

PROPOSITION 4. *Information properties* $(m,n)$ *influence the volatilities of inflation and output as follows:*

*(i) Volatilities increase strictly in* $n$*, i.e., more agreement always increases volatilities.*

*(ii) There exists a unique* $\hat{n}$*, such that volatilities decrease strictly in* $m$ *if and only if* $n < \hat{n}$*, i.e., more accurate average forecast decreases volatilities when disagreement is sufficiently high.*

Proposition 4 suggests that holding fixed the average forecast-error volatility ($m$), a higher degree of agreement among agents (increasing $n$) leads to higher fluctuations in output and inflation. On the other hand, holding the degree of disagreement fixed, reducing the size of the average forecast error has a non-monotonic effect on the volatility. We find that increasing $m$ helps to stabilize inflation and output if and only if the disagreement among the firms is sufficiently high.

The intuition for these results is due to a trade-off between the *noise-diminishing* and the *sensitivity* effects. Specifically, as we explained earlier, the lack of commitment by the central bank induces an implicit coordination motive among the firms, making it necessary for an individual firm to forecast the forecasts of others. That is, in forming its best forecast of the future inflation, a firm uses its information to estimate not only the central bank's inflation incentive but also others' beliefs about the incentive. We call the first use of information the *fundamental* value of information and the second use the *strategic* value of information. Under the information structure specified in our model, improvements in the precision of the average forecast $m$ and the agreement $n$ play different roles in affecting the two uses of information (see Liang and Zhang 2019). First, increasing either $m$ or $n$ diminishes the size of (common or idiosyncratic) noises and moves the firms' signals closer to the central bank's true target, which enhances the fundamental value of information. Second, increasing $n$ increases the strategic value of information, while increasing $m$ decreases the strategic value of information. This is because the strategic value of information is determined by the correlation between firms' private signals, $corr\left(s_{\tau}^{i}, s_{\tau}^{i'}\right) = \frac{\frac{1}{m}+\frac{1}{q}}{\frac{1}{m}+\frac{1}{n}+\frac{1}{q}}$ for $i \neq i'$, which is strictly increasing in $n$ but decreasing in $m$. Intuitively, increasing $m$ reduces the size of common noises and hence the common variation among the firms' signals, reducing the correlation between the signals, while increasing $n$ decreases the size of idiosyncratic noises and hence the idiosyncratic variation, increasing the correlation.

The different role of $m$ and $n$ in influencing the value of information determines their effects on the volatilities. We first explain the effect of higher agreement. Since increasing $n$ (higher agreement) increases both the fundamental and the strategic value of the information, all firms respond more sensitively to their signals $\left\{s_{t+1}^{i}, s_{t+2}^{i}\right\}$ in forming their forecasts ($W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$ both increase). After the idiosyncratic noises $\left\{\varepsilon_{t+1}^{i}, \varepsilon_{t+2}^{i}\right\}$ are diversified away in the aggregation, the average expectation of the firms becomes more responsive to the average signals $\{\bar{s}_{t+1}, \bar{s}_{t+2}\}$. This is the sensitivity effect of increasing $n$. When the central bank cannot commit, it is tempted to respond more to the aggregate expectation, making its monetary policy more sensitive to the errors in firms' average expectation as well. As a result, the equilibrium inflation induced by the monetary policy is driven by the errors in the aggregate expectation to a larger extent and becomes more volatile.

The effect of increasing $m$ differs from that of increasing $n$ in two ways. First, increasing $m$ increases the fundamental value of the information but decreases the strategic value. Overall, increasing $m$ still increases the firms' sensitivity to their signals (increases $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$). The higher sensitivity leads to higher volatilities through the transmission mechanism illustrated above; however, this sensitivity effect of $m$ is weaker than that of $n$ because the decrease in the strategic value of the information led by higher $m$ dampens the increase in the sensitivity. Second, in aggregating the firms' forecasts, the common noise $\{\eta_{t+1}, \eta_{t+2}\}$ is not diversified away as the idiosyncratic noises $\left\{\varepsilon^i_{t+1}, \varepsilon^i_{t+2}\right\}$. This captures the noise-diminishing effect of increasing $m$: a higher $m$ directly diminishes the size of the common noise and makes the average expectation and hence the inflation rate less volatile. The net effect of $m$ on the volatilities thus depends on the trade-off between the sensitivity effect and the noise-diminishing effect. When the disagreement is sufficiently high, the strategic value of information in forecasting the forecasts of others becomes important. Due to the adverse effect of $m$ on the strategic value, the firms are more reluctant to respond to their information, despite the fact that the increase in $m$ improves the fundamental value. As a result, the sensitivity effect becomes weak and dominated by the noise-diminishing effect. Accordingly, increasing $m$ leads to lower volatilities. Otherwise, when the disagreement is low, the strategic value of information becomes less important, making the sensitivity effect strong and dominate the noise-diminishing effect. In these cases, increasing $m$ amplifies the volatilities.

## 5. Additional Analysis

In this section, we derive some additional results to enrich the implications of our paper.[15]

[15]We thank two annoymous reviewers for suggesting this additional analysis, which helps to enrich the implications of our paper.

### *5.1 Fixing the Total Precision of Firms' Signals*

In our main analysis, we have analyzed the effects of separately varying the accuracy of firms' average forecast $m$ and the agreement among firms' forecasts $n$. It is also interesting to examine how the volatilities change with either $m$ or $n$, fixing the total precision of firms' signals $\frac{1}{m} + \frac{1}{n}$. In this case, the precision of the first-order expectation stays constant, so any variations in the volatilities are triggered by changes in the higher-order expectations. We summarize our results in the following proposition.

PROPOSITION 5. *Fixing the total precision of firms' signals* $\frac{1}{m} + \frac{1}{n}$*, information properties* $(m, n)$ *influence the volatilities of inflation and output as follows:*

*(i) Volatilities increase strictly in* $n$*, i.e., more agreement always increases volatilities.*

*(ii) Volatilities decrease strictly in* $m$*, i.e., more accurate average forecast always decreases volatilities.*

Proposition 5 suggests that, holding the total precision of firms' signals constant, a higher degree of agreements among firms still magnifies the volatilities in output and inflation, similar to the result in Proposition 4. However, while Proposition 4 points to a non-monotonic effect of changing the accuracy of the average forecast on the volatilities, Proposition 5 shows that, when the total precision is fixed, reducing the average forecast error always helps to stabilize inflation and output. The intuition for this result, again, lies in how varying the degree of agreement $n$ and the average forecast accuracy $m$ affects the fundamental and the strategic value of firms' signals. First, since the total precision of firms' signals is fixed, changing either $n$ or $m$ will not alter the signals' fundamental value. Second, recall that the strategic value of the signals is determined by the correlation between firms' private signals, $corr\left(s_\tau^i, s_\tau^{i'}\right) = \frac{\frac{1}{m}+\frac{1}{q}}{\frac{1}{m}+\frac{1}{n}+\frac{1}{q}}$ for $i \neq i'$. Fixing the total precision $\frac{1}{m} + \frac{1}{n}$, the correlation is strictly increasing in $n$ and decreasing in $m$. Accordingly, a higher agreement $n$ improves the strategic value of firms' signals and makes all firms respond more sensitively to their signals, which, in turn, contributes to higher fluctuations in output and inflation through the transmission mechanism discussed in our main analysis. Conversely, a higher average forecast precision $m$ impairs the strategic value of firms' signals and makes all firms respond less sensitively to their signals and the noises in the signals. In addition, a higher $m$ also diminishes the size of the common noise. Combining the two effects, improving the accuracy of firms' average forecasts helps to mitigate volatilities in output and inflation.

To illustrate the implications of Proposition 5, consider a shock to the information environment such that after the shock, the disagreement among firms vanishes (i.e., the idiosyncratic shock in the signal $\varepsilon^i_{t+j}$ is muted, $n = \infty$) but the total precision of firms' signals is unaffected. Note that from Proposition 5, since increasing the agreement always amplifies volatilities, imposing full agreement would in fact result in maximal fluctuations in the economy among all scenarios with the same level of total precision. Stated differently, our analysis cautions against efforts to shrink disagreement/dispersion in market participants' understanding of central banks' operations, even if these efforts do not reduce market participants' total knowledge about central banks.

### *5.2 Fundamental and Non-fundamental Volatilities*

In our main analysis, we have examined how varying the informational properties $\{m, n\}$ affects the volatilities in output and inflation. Examining Equation (34) suggests that the properties $m$ and $n$ can influence the volatilities through two components: (i) the fundamental volatility stemming from shocks to the central bank's inflation incentives (i.e., the third term in (34)) and (ii) the non-fundamental volatility from the common noises in firms' signals (i.e., the fourth term in (34)). To illustrate the underlying economic forces of our paper, it is helpful to decompose the effects of $\{m, n\}$ on the fundamental and non-fundamental volatilities. The following proposition summarizes our analysis of such decomposition.

PROPOSITION 6. *Information properties $(m, n)$ influence the fundamental and non-fundamental volatilities of inflation and output as follows:*

*(i) Both the fundamental and non-fundamental volatilities increase strictly in $n$, i.e., more agreement always increases both volatilities.*

*(ii) The fundamental volatilities increase strictly in $m$, i.e., more accurate average forecast always increases the fundamental volatilities.*

*(iii) There exists a threshold $\hat{n}'$, such that the non-fundamental volatilities decrease strictly in $m$ if and only if $n < \hat{n}'$, i.e., more accurate average forecast decreases the non-fundamental volatilities when disagreement is sufficiently high.*

The message of Proposition 6 echoes our main result in Proposition 4. In particular, recall from our discussion of Proposition 4 that increasing the agreement always makes firms more responsive to their signals. Since the signals commingle the fundamental shocks to the central bank's inflation incentives with some noises, firms also react more sensitively to both the fundamental shocks and the non-fundamental noises. Through the transmission mechanism discussed previously, firms' higher sensitivities lead to heightened fundamental and non-fundamental volatilities. This explains part (i) of Proposition 6.

Similarly, recall that although increasing the average forecast precision $m$ has conflicting effects on the fundamental and the strategic value of firms' information, its overall effect on firms' sensitivity to their signals is still positive. This, in turn, contributes to higher fundamental volatilities. This explains part (ii) of Proposition 6.

The last part of Proposition 6 is also in line with Proposition 4. Note that a reduction of the average forecast error (a higher $m$) affects the non-fundamental volatilities in two ways: it not only increases firms' sensitivity to their signals but also diminishes the size of the common noises. From the discussion of Proposition 4, the noise-diminishing effect dominates the sensitivity effect when the disagreement is high. Accordingly, in these cases, increasing $m$ helps to reduce the non-fundamental volatilities.

### *5.3 Adjusting the Central Bank's Objective Function*

A main takeaway from our analysis is that the informational frictions faced by firms can potentially lead to amplified volatilities in inflation and output. These volatilities are driven by the central bank's equilibrium choice of the discretionary monetary rule, and hence depend on the central bank's objective function (8). Accordingly, one may argue that, to mitigate these aggregate volatilities, the central bank ex ante may have incentives to adjust its objective function. While a full characterization of the central bank's optimal objective function is beyond the scope of our paper, we now explore a specific way of adjusting the objective function, that is, the central bank adjusting the relative weight $\alpha$ placed on output deviations $(x_t - k_t)^2$. In practice, a lower $\alpha$ can be interpreted as appointing a more conservative central banker (Rogoff 1985; Clarida, Gali, and Gertler 1999). We summarize the effect of the weight $\alpha$ on the inflation volatility in the proposition below.

PROPOSITION 7. *The inflation volatility increases strictly in the weight $\alpha$ on output deviations.*

Proposition 7 suggests that the informational frictions faced by firms induce greater inflation volatilities if the central bank is more concerned about the output gap target. Stated differently, to stabilize aggregate fluctuations, the central bank should reduce the weight placed on output deviations. To see the intuition, recall from Equation (21) that the equilibrium inflation is more sensitive to the terms of higher-order beliefs if the central bank is more responsive to changes in average expectations of inflation in choosing its discretionary monetary policy, as captured by the response coefficient $\frac{\alpha\beta}{\alpha+\lambda^2}$. Note that this coefficient is strictly increasing in the weight $\alpha$. A central bank focusing more on output gaps is more tempted to push inflation $\pi_t$ higher in order to lower the output gap. This, in turn, sets the trap for the central bank to respond to firms' inflation expectations, thus amplifying the inflation volatility through the coordination channel identified in our main analysis.

### *5.4 A Microfoundation of the Common Noise in Firms' Signals*

Our analysis suggests that the common noise in firms' signals $\eta_t$ plays a crucial role, as it contributes to the non-fundamental uncertainties in the output and inflation. In our main model, we do not specify how the common noise may arise in firms' information environment. We study an extension below to offer a microfoundation in which firms learn the central bank's policy objective from an endogenous variable and such learning is subject to a common noise. For simplicity, in this extension, we adopt the macroeconomic framework in Barro and Gordon (1983a) and capture endogenous learning following Stein and Sunderam (2018). To illustrate the main idea, we only consider the static version of the learning model in Stein and Sunderam (2018, Section II); Stein and Sunderam (2018, Section IV) also extend the learning model to a fully dynamic model.

Specifically, the operation of the economy is described by a "Phillips curve" (Equation 1 on pp. 592, Barro and Gordon 1983a):

$$U_t = U_t^n - a\left(\pi_t - \pi_t^e\right), \tag{35}$$

where $U_t$ denotes the unemployment rate and is a proxy for the overall state of real activity, $U_t^n$ denotes the natural unemployment rate, $\pi_t$ denotes the inflation rate, and $\pi_t^e$ denotes firms' expected inflation. The coefficient $a > 0$ represents the "Phillips curve slope." The central bank's objective is to minimize a social loss function (Barro and Gordon 1983a, Equation (3) on p. 593):

$$Z_t = E\left[\left(U_t - k_t\right)^2\right] + bE\left[\pi_t^2\right], \tag{36}$$

where $k_t \sim N\left(U_t^n, \frac{1}{\tau_k}\right)$ represents the central bank's preferred unemployment rate target, as discussed in Barro and Gordon (1983a). $k_t$ can be either higher or lower than the natural unemployment rate, depending on the central bank's target. The parameter $b > 0$ captures the central bank's relative weight on minimizing inflation in its objective function. The same as in Stein and Sunderam (2018), we assume that $k_t$ is the central bank's private information and unknown to firms at the beginning of period $t$. Specifically, we assume that the central bank sets the inflation rate $\pi_t$ by following a partial adjustment rule of the form

$$\pi_t = \mu (k_t - U_t^n) + \varepsilon_t, \tag{37}$$

where $\varepsilon_t \sim N\left(0, \frac{1}{\tau_\varepsilon}\right)$ represents a noise "that is overlaid onto the rate-setting process" by Stein and Sunderam (2018, p. 1024), who interpret the noise as either a "tremble" in the central bank's optimal choice of $\pi_t$ or "coming from the Fed's use of round numbers (typically in 25 bps) for the funds rate settings that it communicates to the market," whereas its private information about $k_t$ is presumably continuous. As we will show soon, the noise $\varepsilon_t$ generates a common noise in the endogenous signal learned by firms and prevents firms from fully recovering the central bank's private information $k_t$. The parameter $\mu$ is the central bank's response coefficient to its unemployment target, and the central bank will set $\mu$ optimally in equilibrium to minimize the social loss function $Z_t$.

Firms try to infer the central bank's private information $k_t$ based on their observation of the aggregate state of the economy, proxied by the unemployment rate $U_t$. To do so, firms conjecture that in equilibrium, the unemployment rate and the inflation rate take the following forms:

$$U_t = U_t^n + \hat{\delta} (k_t - U_t^n) - \hat{\lambda} \varepsilon_t, \tag{38}$$

$$\pi_t = \hat{\mu} (k_t - U_t^n) + \varepsilon_t. \tag{39}$$

That is, firms correctly conjecture the equilibrium forms of the unemployment rate and the inflation rate but do not observe the sets of actual response coefficients. Instead, firms conjecture the values of these coefficients as $\left\{\hat{\delta}, \hat{\lambda}, \hat{\mu}\right\}$, where $\hat{x}$ denotes firms' conjecture of variable $x$. Rational expectations require that in equilibrium, firms' conjectures are correct and coincide with the equilibrium values (yet unable to fully invert, similar to Stein and Sunderam 2018). Importantly, note that the unemployment rate $U_t$ is an endogenous signal about the central bank's private information subject to a common noise. The noise arises because of the noise $\varepsilon_t$ in the central bank's rate-setting process in Equation (37), which, in turn, enters the unemployment rate through the Phillips curve in Equation (35).

We will verify that $\lambda > 0$ so there is indeed a common noise term in the endogenous signal $U_t$.

Using their conjectured form of the unemployment rate in Equation (38), firms infer imperfectly the central bank's private information $k_t$ from their observation of $U_t$. Rewriting (38) yields

$$\hat{U}_t \equiv U_t^n + \frac{U_t - U_t^n}{\hat{\delta}} = k_t - \frac{\hat{\lambda}}{\hat{\delta}}\varepsilon_t. \quad (40)$$

$\hat{U}_t$ can be viewed as an adjusted unemployment rate that constitutes an endogenous signal of $k_t$, where the precision of $\hat{U}_t$ is $\frac{\hat{\delta}^2}{\hat{\lambda}^2}\tau_\varepsilon$. Standard Bayesian updating gives

$$E\left[k_t|\hat{U}_t\right] = \frac{\frac{\hat{\delta}^2}{\hat{\lambda}^2}\tau_\varepsilon}{\frac{\hat{\delta}^2}{\hat{\lambda}^2}\tau_\varepsilon + \tau_k}\hat{U}_t + \frac{\tau_k}{\frac{\hat{\delta}^2}{\hat{\lambda}^2}\tau_\varepsilon + \tau_k}U_t^n = U_t^n + \chi\left(U_t - U_t^n\right), \quad (41)$$

where $\chi \equiv \frac{\hat{\delta}\tau_\varepsilon}{\hat{\delta}^2\tau_\varepsilon + \hat{\lambda}^2\tau_k}$. Intuitively, when there is less noise in the unemployment rate $U_t$ (i.e., a smaller $\hat{\lambda}$), firms learn more information about $k_t$ from $U_t$ and hence react more to a change in $U_t$ (i.e., a higher $\chi$).

Given firms' inference of $k_t$ and the conjectured form of the equilibrium inflation rate in Equation (39), firms also form an expectation of inflation:

$$\pi^e = E\left[\pi_t|U_t\right] = \hat{\mu}\left(E\left[k_t|U_t\right] - U_t^n\right) = \hat{\mu}\chi\left(U_t - U_t^n\right). \quad (42)$$

Substituting Equation (42) into Equation (35) solves the equilibrium unemployment rate:

$$U_t = U_t^n - \frac{a\pi_t}{1 - a\hat{\mu}\chi}. \quad (43)$$

Given the central bank's choice of inflation rate in Equation (37) and the equilibrium unemployment rate in Equation (43), the central bank's objective function $Z_t$ in Equation (36), taking expectations over the noise $\varepsilon_t$, can be written as

$$Z_t = \left(1 + \frac{a\mu}{1 - a\hat{\mu}\chi}\right)^2 \frac{1}{\tau_k} + \left(\frac{a}{1 - a\hat{\mu}\chi}\right)\frac{1}{\tau_\varepsilon} + b\left(\frac{\mu^2}{\tau_k} + \frac{1}{\tau_\varepsilon}\right). \quad (44)$$

The central bank minimizes $Z_t$ by choosing the optimal response coefficient $\mu$ in Equation (37). The central bank does so taking as given firms' conjectures $\left\{\hat{\delta}, \hat{\lambda}, \hat{\mu}\right\}$. Taking the first-order condition with respect to $\mu$ gives

$$\left(1 + \frac{a\mu}{1 - a\hat{\mu}\chi}\right)\frac{a}{1 - a\hat{\mu}\chi} + b\mu = 0. \tag{45}$$

In the rational expectations equilibrium, firms' conjectures are correct, i.e., $\hat{\mu} = \mu$, $\hat{\delta} = \delta$, and $\hat{\lambda} = \lambda$. Imposing these requirements in the first-order condition (45) and matching the coefficients in the equilibrium unemployment rate in Equation (43) with firms' conjecture in Equation (38) determine the sets of coefficients, $\{\delta, \lambda, \mu\}$, in equilibrium. We summarize the equilibrium in the following proposition.

PROPOSITION 8. *Consider a model in which firms learn about the central bank's private information from an endogenous variable. The equilibrium unemployment rate and inflation take the following form:*

$$U_t = U_t^n + \delta\left(k_t - U_t^n\right) - \lambda\varepsilon_t, \tag{46}$$

$$\pi_t = \mu\left(k_t - U_t^n\right) + \varepsilon_t, \tag{47}$$

*where the coefficients are given by*

$$\delta = \frac{a\sqrt{b\left(1-\delta\right)\delta\tau_k}}{\left(1-\delta\right)\delta\tau_\varepsilon + b\tau_k} \in \left(0,1\right), \tag{48}$$

$$\mu = -\sqrt{\frac{1}{b}\left(1-\delta\right)\delta} < 0, \tag{49}$$

$$\lambda = \sqrt{\frac{b\delta}{1-\delta}} > 0. \tag{50}$$

Proposition 8 confirms that in equilibrium, since $\lambda > 0$, the unemployment rate $U_t$ indeed constitutes an endogenous signal about the central bank's unemployment rate target $k_t$, subject to the common noise $\varepsilon_t$ stemming from the rate-setting process. The unemployment rate $U_t$ is informative about $k_t$ because the central bank, facing the downward-sloping Phillips curve, is tempted to inject inflation in order to reduce the unemployment rate toward its preferred target $k_t$ (i.e., $\mu < 0$). Accordingly, both the equilibrium inflation rate and the equilibrium unemployment rate become dependent on $k_t$ so that observing the unemployment rate reveals information about $k_t$. This model extension establishes the endogenous source of the common noise in the private signals exogenously specified in the base model.

### *5.5 A Microfounded Phillips Curve*

In our main analysis, to maintain tractability, we have assumed a reduced-form Phillips curve (2) in line with the standard New Keynesian Phillips curve when information is complete. Nonetheless, recent work by Angeletos and Lian (2018) and Angeletos and Huo (2021) suggest that when information is incomplete, the form of the Phillips curve varies from the standard one. Importantly, both studies show that, considering incomplete information, the current inflation depends on the entire future path of average expectations about the output gap and the inflation rate, instead of only the average expectation about the next-period inflation. Angeletos and Huo (2021, p. 1174) demonstrate that analyzing the equilibrium under the modified Phillips curve is considerably more complicated, as "the relevant set of higher-order beliefs is significantly richer." In light of this complexity, we have imposed a simple reduced-form Phillips curve in our main analysis to focus on deriving implications for how central bank transparency/disagreement affects aggregate volatilities. To assess the robustness of our analysis, we now analyze a variant of our model with a microfounded Phillips curve.

Specifically, consider a setting in which the Calvo (1983) friction lasts only for one period, i.e., a firm that has reset its price $p_t^j$ in period $t$ is restricted from resetting its price in period $t + 1$ with probability $\theta \in (0, 1)$, whereas a firm restricted from resetting its price in period $t$ gains full price-resetting flexibility in period $t + 1$. For simplicity, in this extension, we assume that the cost-push shocks $u_t = 0$. Consider a firm $j$ that has the opportunity to reset its price in period $t$. Following similar steps as in Angeletos and Lian (2018, Equation (32)), the optimal reset price, denoted by $p_t^{j*}$, can be derived as

$$p_t^{j*} = \frac{1}{1+\beta\theta}\left(mc_t^j + p_t\right) + \frac{\beta\theta}{1+\beta\theta}E_t^j\left[mc_{t+1}^j + p_{t+1}\right], \tag{51}$$

where $mc_t^j$ denotes firm $j$'s real marginal cost in period $t$. Conditional on the firm being restricted from resetting the price in period $t+1$, it gains full price-resetting flexibility in period $t+2$. Accordingly, the optimal reset price $p_t^{j*}$ only depends on the firm's expectation about the marginal cost and the aggregate price in period $t+1$. Aggregating $p_t^{j*}$ over the population of the firms gives

$$p_t^{*} = \frac{1}{1+\beta\theta}\left(mc_t + p_t\right) + \frac{\beta\theta}{1+\beta\theta}\bar{E}_t\left[mc_{t+1} + p_{t+1}\right], \tag{52}$$

where $\bar{E}_t[\cdot]$ denotes the average expectation of the firms. As shown in the proof of Proposition 9, at the steady state, in each period $t$, a fraction $\frac{1}{1+\theta}$ of the firms can reset the price, whereas the remaining firms are restricted from resetting the price. This, in turn, gives the inflation rate:

$$\pi_t = \frac{p_t^* - p_{t-1}}{1+\theta}. \tag{53}$$

Using (52) and (53) and applying the usual condition that the real marginal cost is proportional to the output gap (Clarida, Gali, and Gertler 1999), i.e., $mc_t = \kappa x_t$, we obtain the following condition for the level of inflation in period $t$:

$$\pi_t = \lambda x_t + \beta' \bar{E}_t\left[\kappa x_{t+1} + \pi_{t+1}\right], \tag{54}$$

where $\beta' \equiv \frac{\beta}{1+\beta+\beta\theta} \in (0,1)$ and $\lambda \equiv \frac{\kappa}{\beta\theta+\theta(1+\beta\theta)} > 0$. Note that (54) is identical to the reduced-form Phillips curve (2) assumed in the main analysis except that in the microfounded Phillips curve (54), the inflation rate $\pi_t$ also depends on the expectation about the future output gap in period $t+1$. To assess how this change affects the robustness of our analysis, we now derive the equilibrium inflation rate $\pi_t^{**}$. Note that under the modified Phillips curve, the optimal discretionary monetary policy remains intact, i.e.,

$$x_t = -\frac{\lambda}{\alpha}\pi_t + k_t, \tag{55}$$

because the central bank still takes the firms' future expectations about $x_{t+1}$ and $\pi_{t+1}$ as given. Substituting the optimal discretionary monetary policy into the Phillips curve (54) gives

$$\pi_t = \frac{\alpha\lambda}{\alpha+\lambda^2}k_t + \frac{\alpha\beta'\kappa}{\alpha+\lambda^2}\bar{E}_t\left[k_{t+1}\right] + \frac{\left(\alpha-\lambda\kappa\right)\beta'}{\alpha+\lambda^2}\bar{E}_t\left[\pi_{t+1}\right]. \quad (56)$$

Comparing the modified law of motion (56) for the inflation $\pi_t$ with its counterpart (17) in the main analysis yields two insights. First, the inflation $\pi_t$ depends not only on the central bank's inflation incentive $k_t$ in period $t$ but also on average expectations about the future inflation incentive $k_{t+1}$. The latter result is driven by the modified Phillips curve (54), where the inflation $\pi_t$ depends on average expectations about the future output gap $x_{t+1}$ and, accordingly, the central bank's future inflation incentive $k_{t+1}$ due to the discretionary rule (55). Second, the inflation $\pi_t$ continues to depend on average expectations $\bar{E}_t\left[\pi_{t+1}\right]$, although the coefficient before $\bar{E}_t\left[\pi_{t+1}\right]$, $\frac{\beta'(\alpha-\lambda\kappa)}{\alpha+\lambda^2}$, can be negative. The reason is that when the expectation about the future inflation $\pi_{t+1}$ increases, firms rationally anticipate that the central bank will choose a policy in period $t+1$ to reduce the future output gap $x_{t+1}$. Such policy, in turn, dampens the current inflation through the Phillips curve (54). This economic force goes against the usual force that the central bank responds positively to changes in $\bar{E}_t\left[\pi_{t+1}\right]$ in setting $\pi_t$, and can even make $\pi_t$ respond negatively to $\bar{E}_t\left[\pi_{t+1}\right]$ when $\pi_t$ is more dependent on the future output gap $x_{t+1}$ (i.e., $\lambda\kappa = \frac{\kappa^2}{\beta\theta+\theta(1+\beta\theta)} > \alpha$). We show that, as long as $\kappa$ is sufficiently small so that $\pi_t$ responds positively to $\bar{E}_t\left[\pi_{t+1}\right]$, all the implications from our main analysis remain valid under the modified Phillips curve. To see this, iterating (56) gives

$$\pi_t^{**} = \frac{\alpha\lambda}{\alpha+\lambda^2}k_t + \left\{\sum_{l=1}^{\infty}\left(\frac{\left(\alpha-\lambda\kappa\right)\beta'}{\alpha+\lambda^2}\right)^{l-1}\frac{\alpha\beta'}{\alpha+\lambda^2}\frac{\alpha\left(\lambda+\kappa\right)}{\alpha+\lambda^2}\bar{E}_t^l\left[k_{t+l}\right]\right\}. \quad (57)$$

Under the modified Phillips curve (54), the equilibrium inflation rate $\pi_t^{**}$ continues to be a function of the sum of the higher-order beliefs about $\{k_{t+l}\}_{l=0}^{l=\infty}$, similar to that characterized in Proposition 1. The only difference is that the "discounting factor" before

$\bar{E}_t^l\left[k_{t+l}\right]$ is $\frac{(\alpha-\lambda\kappa)\beta'}{\alpha+\lambda^2}$ instead of $\frac{\alpha\beta}{\alpha+\lambda^2}$. Accordingly, we show that as long as $\frac{(\alpha-\lambda\kappa)\beta'}{\alpha+\lambda^2} > 0$, the implications regarding the effect of the informational properties on the volatilities of inflation and output are similar to the ones in our main analysis. We formally state these results in the following proposition.

PROPOSITION 9. *Under the modified Phillips curve (54), the equilibrium inflation rate $\pi_t^{**}$ depends on the sum of the higher-order beliefs about $\{k_{t+l}\}_{l=0}^{l=\infty}$, i.e.,*

$$\pi_t^{**} = \frac{\alpha\lambda}{\alpha+\lambda^2} k_t + \left\{ \sum_{l=1}^{\infty} \left( \frac{\beta'\left(\alpha-\lambda\kappa\right)}{\alpha+\lambda^2} \right)^{l-1} \frac{\alpha\beta'}{\alpha+\lambda^2} \frac{\alpha\left(\lambda+\kappa\right)}{\alpha+\lambda^2} \bar{E}_t^l\left[k_{t+l}\right] \right\}. \tag{58}$$

*If $\frac{\kappa^2}{\beta\theta+\theta(1+\beta\theta)} < \alpha$, information properties $(m, n)$ influence the volatilities of inflation and output as follows:*

*(i) Volatilities increase strictly in $n$, i.e., more agreement always increases volatilities.*

*(ii) There exists a unique $\hat{n}''$, such that volatilities decrease strictly in $m$ if and only if $n < \hat{n}''$, i.e., more accurate average forecast decreases volatilities when disagreement is sufficiently high.*

## 6. Conclusion

With its simplicity, our paper makes a core argument for the inclusion of information diversity among agents in monetary policy discussions. A direct implication of our model is on the explanation and characterization of the observed inflation dynamics. Our model would suggest that the precision of the aggregate estimation of future inflation is a determinant of current inflation. In this regard, our paper is related to the voluminous macroliterature on inflation trend (Goodfriend and King 2012 and Ascari and Sbordone 2014). In these studies, firms are more sophisticated in their understanding of the inflation trend and adjust their pricing behavior (such as indexing). In an extension, we verify that our main qualitative results survive in a more general model (e.g., Woodford 2008) in which an inflation trend term is inserted into the New Keynesian Phillips curve.

More broadly, we view our paper as an attempt at constructing a positive understanding of the macroeconomy under the information imperfections about the incentives of an authority player. Our paper is not directly concerned about how these imperfections emerge endogenously from the *information production* of each player in the model, but any such studies should take into account the results of our paper. Recent interests in studying the communication strategies of the central bank are evidence of its perceived importance (see, e.g., Rudebusch and Williams 2008).

Aside from the information flow from the central bank to the marketplace, a more organic environment would also feature active private information activities. As shown in the Fed-watch literature, individual agents are motivated to acquire relevant information in anticipated information management by the central bank.

Finally, our paper raises issues that future studies could blend with other important considerations related to information and coordination. They include other coordination problems in macroeconomics (Cooper and John 1988; Kiyotaki and Wright 1989; Baxter and King 1991), robust policies by Hansen and Sargent (2007), and the information role of the financial market (King 1982; Baxter and King 1991).

## Appendix. Proofs

### *Proof of Proposition 1*

This can be verified by iterating (17).

### *Proof of Proposition 2*

For our convenience, we define the vectors of firm $i$'s demeaned signals at period $t$ and the vector of the demeaned average signals at period $t$ as

$$S_t^i = \begin{bmatrix} k_t - \bar{k} \\ s_{t+1}^i - \bar{k} \\ s_{t+2}^i - \bar{k} \end{bmatrix}, \bar{S}_t = \begin{bmatrix} k_t - \bar{k} \\ \bar{s}_{t+1} - \bar{k} \\ \bar{s}_{t+2} - \bar{k} \end{bmatrix}, \tag{A.1}$$

the variance of $S_t^i$ as

$$Var\left(S_t^i\right) = \Sigma = \begin{bmatrix} \frac{1}{q_k} & \frac{\rho_k}{q_k} & \frac{\rho_k^2}{q_k} \\ \frac{\rho_k}{q_k} & \frac{1}{q_k} + \frac{1}{m} + \frac{1}{n} & \frac{\rho_k}{q_k} \\ \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} + \frac{1}{m} + \frac{1}{n} \end{bmatrix}, \tag{A.2}$$

and the covariance between $\bar{S}_{t+1}$ and $S_t^i$ as

$$Cov\left(\bar{S}_{t+1}, S_t^i\right) = \Omega = \begin{bmatrix} \frac{\rho_k}{q_k} & \frac{1}{q_k} & \frac{\rho_k}{q_k} \\ \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} + \frac{1}{m} \\ \frac{\rho_k^3}{q_k} & \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} \end{bmatrix}. \tag{A.3}$$

In particular, we define the first row of $\Omega$, the covariance between $k_{t+1}$ and $S_t^i$, as

$$\Omega^{\text{row1}} = L = \begin{bmatrix} \frac{\rho_k}{q_k} & \frac{1}{q_k} & \frac{\rho_k}{q_k} \end{bmatrix}. \tag{A.4}$$

We now derive the hierarchy of higher-order beliefs. In the first-order belief in period $t$, each firm's forecast of $k_{t+1}$ is

$$E_t^i\left[k_{t+1}\right] = \bar{k} + L\Sigma^{-1}S_t^i, \tag{A.5}$$

and the average forecast is

$$\bar{E}_t\left[k_{t+1}\right] = \bar{k} + L\Sigma^{-1}\bar{S}_t. \tag{A.6}$$

Building on the first-order expectation, now move to the second-order belief. For firm $i$, its period-$t$ belief about the aggregate period $t+1$ belief about the central bank's period $t+2$ incentive becomes

$$E_t^i\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right] = \bar{k} + E_t^i\left[L\Sigma^{-1}\bar{S}_{t+1}\right] = \bar{k} + L\Sigma^{-1}E_t^i\left[\bar{S}_{t+1}\right], \tag{A.7}$$

where $E_t^i\left[\bar{S}_{t+1}\right] = \Omega\Sigma^{-1}S_t^i$. Therefore, the average second-order belief becomes

$$\bar{E}_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right] = \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}\bar{S}_t. \tag{A.8}$$

Notice that the law of iterated expectation fails, i.e., $\bar{E}_t^2\left[k_{t+2}\right] \neq \bar{E}_t\left[k_{t+2}\right] = \bar{k} + \begin{bmatrix} \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} \end{bmatrix} \Sigma^{-1}\bar{S}_t$, since $L\Sigma^{-1}\Omega = \begin{bmatrix} \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} \frac{\left(\frac{1}{m}+\frac{1}{q}\right)\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)+\left(\frac{1}{mq}+\frac{1}{mn}+\frac{2}{nq}+\frac{1}{n^2}\right)\rho_k^2}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2} \end{bmatrix} \neq \begin{bmatrix} \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} \end{bmatrix}$ for $n \neq \infty$ (i.e., there is some disagreement among firms). In particular, we verify that in $\bar{E}_t^2\left[k_{t+2}\right]$, the signal $\bar{s}_{t+2}$ is weighted less than in $\bar{E}_t\left[k_{t+2}\right]$. Moreover, for the third-order belief, firm $i$'s period-$t$ belief about the aggregate period $t+1$ belief about the aggregate period $t+2$ belief about the central bank's period $t+3$ incentive becomes

$$\begin{aligned} E_t^i\left[\bar{E}_{t+1}\left[\bar{E}_{t+2}\left[k_{t+3}\right]\right]\right] &= \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}E_t^i\left[\bar{S}_{t+1}\right] \\ &= \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}\Omega\Sigma^{-1}S_t^i, \end{aligned} \tag{A.9}$$

and thus the average third-order belief becomes

$$\begin{aligned} \bar{E}_t\left[\bar{E}_{t+1}\left[\bar{E}_{t+2}\left[k_{t+3}\right]\right]\right] &= \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}\Omega\Sigma^{-1}\bar{S}_t \\ &= \bar{k} + L\Sigma^{-1}\left(\Omega\Sigma^{-1}\right)^2\bar{S}_t. \end{aligned} \tag{A.10}$$

Keeping iterating $\bar{E}_t^3\left[k_{t+3}\right]$ characterizes the entire hierarchy of higher-order beliefs with

$$\bar{E}_t^l\left[k_{t+l}\right] = \bar{k} + L\left(\Sigma^{-1}\Omega\right)^{l-1}\Sigma^{-1}\bar{S}_t. \tag{A.11}$$

To derive $\bar{E}_t^l\left[k_{t+l}\right]$, we make an eigenvalue decomposition on $\Sigma^{-1}\Omega$, such that

$$\Sigma^{-1}\Omega = Q\Lambda Q^{-1}, \tag{A.12}$$

where $\Lambda$ is a diagonal matrix with the eigenvalue of $\Sigma^{-1}\Omega$ on its diagonal, i.e.,

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\frac{1}{n}\frac{\rho_k}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2} & 0 \\ 0 & 0 & \rho_k \end{bmatrix}, \tag{A.13}$$

and $Q$ is the associated matrix of eigenvectors. As a result,

$$\bar{E}_t^l\left[k_{t+l}\right]=\bar{k}+LQ\Lambda^{l-1}Q^{-1}\Sigma^{-1}\bar{S}_t$$

$$=\bar{k}+LQ\begin{bmatrix}0 & 0 & 0\\ 0 & \left[\frac{\frac{1}{n}\frac{\rho_k}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}\right]^{l-1} & 0\\ 0 & 0 & \rho_k^{l-1}\end{bmatrix}Q^{-1}\Sigma^{-1}\bar{S}_t, \tag{A.14}$$

and can be simplified into

$$\bar{E}_t^l\left[k_{t+l}\right]=\bar{k}+\rho_k^{l-1}\left\{\left[1-w\left(l\right)\right]\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right]+w\left(l\right)\frac{\bar{s}_{t+2}-\bar{k}}{\rho_k}\right\}, \tag{A.15}$$

where $\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right]=\frac{q}{q+\frac{mn}{m+n}}\rho_k\left(k_t-\bar{k}\right)+\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\bar{s}_{t+1}-\bar{k}\right)$ and

$$w\left(l\right)=\frac{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}$$
$$+\frac{\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)\frac{1}{q}\left[\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)\left(\frac{1}{m}+\frac{1}{q}\right)\right]\times\left\{1-\left[\frac{\frac{1}{n}\frac{1}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}\right]^{l-1}\right\}}{\left[\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2\right]^2\times\left\{1-\left[\frac{\frac{1}{n}\frac{1}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}\right]\right\}}. \tag{A.16}$$

Notice that since $\frac{\frac{1}{n}\frac{1}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}<1$, then $w\left(l\right)$ is strictly increasing in $l$.

*Proof of Proposition 3*

Substituting the expressions for the higher-order-beliefs terms into the expression for the inflation specified in Proposition 1, we have

$$\pi_t^{**} = \frac{\alpha u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right) + \frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right), \tag{A.17}$$

where the "demeaned" higher-order beliefs are

$$\begin{aligned}\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right) &= \left[1-w(l)\right]\left[\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right]-\rho_k\left(k_t-\bar{k}\right)\right] \\ &\quad + w(l)\left(\frac{\bar{s}_{t+2}-\bar{k}}{\rho_k}-\rho_k\left(k_t-\bar{k}\right)\right)\end{aligned} \tag{A.18}$$

with

$$\begin{aligned}\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right]-\rho_k\left(k_t-\bar{k}\right) &= \frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\bar{s}_{t+1}-\bar{k}-\rho_k\left(k_t-\bar{k}\right)\right) \\ &= \frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\eta_{t+1}+\nu_{t+1}\right),\end{aligned} \tag{A.19}$$

and

$$\frac{\bar{s}_{t+2}-\bar{k}}{\rho_k}-\rho_k\left(k_t-\bar{k}\right) = \frac{\nu_{t+2}+\rho_k\nu_{t+1}+\eta_{t+2}}{\rho_k}. \tag{A.20}$$

By the first-order condition, the equilibrium output is

$$
\begin{aligned}
x_t^{**} &= -\frac{\lambda}{\alpha}\pi_t^{**} + k_t \\
&= -\frac{\lambda u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right) \\
&\quad - \frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right). \qquad \text{(A.21)}
\end{aligned}
$$

The equilibrium nominal interest rate can be derived by substituting the pair $(\pi_t^{**}, x_t^{**})$ into the IS curve (7):

$$
i_t^{**} = \frac{E_t^H x_{t+1}^{**}}{\phi} + \frac{g_t}{\phi} - \frac{x_t^{**}}{\phi} + E_t^H \pi_{t+1}^{**}, \qquad \text{(A.22)}
$$

where, given the information set of the household, $I_t^H = \left\{\{u_\tau\}_{\tau=0}^t, \{g_\tau\}_{\tau=0}^t, \{k_\tau\}_{\tau=0}^t\right\}$,

$$
\begin{aligned}
E_t^H x_{t+1}^{**} &= -\frac{\lambda\rho_u u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\lambda^2}\bar{k} \\
&\quad + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\rho_k\left(k_t-\bar{k}\right), \\
E_t^H \pi_{t+1}^{**} &= \frac{\alpha\rho_u u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} \\
&\quad + \frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\rho_k\left(k_t-\bar{k}\right), \qquad \text{(A.23)}
\end{aligned}
$$

and as a result,

$$
\begin{aligned}
i_t^{**} &= \frac{g_t}{\phi} + \frac{u_t}{\alpha(1-\beta\rho_u)+\lambda^2}\left[\alpha\rho_u + \frac{\lambda(1-\rho_u)}{\phi}\right] + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} \\
&\quad + \frac{\alpha\left(k_t-\bar{k}\right)}{\alpha(1-\beta\rho_k)+\lambda^2}\left[\lambda\rho_k - \frac{(1-\beta\rho_k)(1-\rho_k)}{\phi}\right]
\end{aligned}
$$

$$+ \ \frac{1}{\phi}\left[\frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1} \times\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)\right)\right]. \qquad (A.24)$$

*Proof of Proposition 4*

Notice that $x_t^{**}=-\frac{\lambda}{\alpha}\pi_t^{**}+k_t$. Thus

$$Var_t\left(x_t^{**}\right)=\frac{\lambda^2}{\alpha^2}Var_t\left(\pi_t^{**}\right)+Var_t\left(k_t\right)-\frac{2\lambda}{\alpha}Cov_t\left(\pi_t^{**},k_t\right), \quad (A.25)$$

where $Var_t\left(k_t\right)=\frac{1}{q(1-\rho_k)^2}$ and $Cov_t\left(\pi_t^{**},k_t\right)=\frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\frac{1}{q(1-\rho_k)^2}$ are both independent of $m$ and $n$. Therefore, the effects of $(m,n)$ on $Var_t\left(x_t^{**}\right)$ are the same as their effects on $Var_t\left(\pi_t^{**}\right)$.

One can verify $\frac{\partial Var_t(\pi_t^{**})}{\partial n}>0$ by directly computing the derivative. For the sign of $\frac{\partial Var_t(\pi_t^{**})}{\partial m}$, first, one can verify that at $n=0$, $\frac{\partial Var_t(\pi_t^{**})}{\partial m}=0$, $\frac{\partial}{\partial n}\left(\frac{\partial Var_t(\pi_t^{**})}{\partial m}\right)=0$, and $\frac{\partial^2}{\partial n^2}\left(\frac{\partial Var_t(\pi_t^{**})}{\partial m}\right)=-\frac{2\left(1+\rho_k^2\right)}{m^2q^2\left(1-\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\right)^2}<0$. As a result, for $n$ close to 0, $\lim_{n\to 0^+}\frac{\partial Var_t(\pi_t^{**})}{\partial m}<0$. Second, at $n=\infty$,

$$\frac{\partial Var_t\left(\pi_t^{**}\right)}{\partial m} = \frac{\left(1+\left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^2\right)(m+q)^2+2\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\left(q^2-m^2\right) + \left(\left(\frac{\alpha\beta}{\alpha+\lambda^2}m\right)^2+q^2\right)\rho_k^2}{\left(1-\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\right)^2\left[(m+q)^2+mq\rho_k^2\right]^2}>0. \quad (A.26)$$

Therefore, by the intermediate value theorem, there exists an $\hat{n}>0$, such that $\frac{\partial Var_t(\pi_t^{**})}{\partial m}=0$. Lastly, we verify that such an $\hat{n}$ is also unique. More specifically, we verify that $\frac{\partial Var_t(\pi_t^{**})}{\partial m}=0$ can be reduced into $P\left(n\right)=0$ and $P\left(n\right)$ is a fourth-order polynomial of $n$,

$$P\left(n\right)=\kappa_1 n^4+\kappa_2 n^3+\kappa_3 n^2+\kappa_4 n^3+\kappa_5, \qquad (A.27)$$

where the expressions of the coefficients $\{\kappa_i\}_{i=1}^{5}$ are available upon request. We verify that $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_5 < 0$, and the signs of $\kappa_3$ and $\kappa_4$ are ambiguous. However, it is impossible to have $\kappa_3 < 0$ and $\kappa_4 > 0$ at the same time. As a result, there can be the following three possible scenarios of the signs of $\{\kappa_i\}_{i=1}^{5}$:

| $\kappa_1$ | $\kappa_2$ | $\kappa_3$ | $\kappa_4$ | $\kappa_5$ |
|---|---|---|---|---|
| + | + | + | + | − |
| + | + | + | − | − |
| + | + | − | − | − |

,

where "+" means positive and "−" means negative. Notice that for the polynomial $P(n)$, there is one sign change in its coefficients. Therefore, by Descartes's rule of signs, the polynomial $p(n)$ has a unique positive root. That is, there exists a unique $\hat{n}$ that makes $\frac{\partial Var_t(\pi_t^{**})}{\partial m} = 0$. As a result, $\frac{\partial Var_t(\pi_t^{**})}{\partial m} < 0$ if and only if $n < \hat{n}$.

*Proof of Proposition 5*

We only analyze the inflation volatility $Var_t(\pi_t^{**})$, as from Proposition 4 the results regarding the output $Var_t(x_t^{**})$ are the same. We denote the (inverse) total precision $T \equiv \frac{1}{m} + \frac{1}{n}$. Hence $m = \frac{1}{T - \frac{1}{n}}$. Since $m \geq 0$, $n > \frac{1}{T}$. Substituting this into the expression of $Var_t(\pi_t^{**})$ and taking the derivative of $Var_t(\pi_t^{**})$ with respect to $n$ gives

$$\frac{\partial Var_t(\pi_t^{**})}{\partial n} = \frac{Q(n)}{n^2\left(1 - \frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\right)^2\left(n + 2nqT + nq^2T^2 - \frac{\alpha\beta}{\alpha+\lambda^2}q\rho_k + nqT\rho_k^2\right)^3}. \tag{A.28}$$

The denominator is positive because $2nqT - \frac{\alpha\beta}{\alpha+\lambda^2}q\rho_k > 2nqT - q\rho_k > 2q - q\rho_k > 0$. The first step uses $\frac{\alpha\beta}{\alpha+\lambda^2} < 1$, the second step uses $\frac{1}{n} < T$, and the last step uses $\rho_k < 1$. The numerator $Q(n)$ is a third-order polynomial of $n$. We now prove that for $n > \frac{1}{T}$,

$Q(n) > 0$. First, replacing $n$ in $Q(n)$ with $n = b + \frac{1}{T}$ (where $b > 0$), we obtain

$$Q(b) = \delta_1 b^3 + \delta_2 b^2 + \delta_3 b + \delta_4, \tag{A.29}$$

where the expressions of the coefficients $\{\delta_i\}_{i=1}^4$ are available upon request. Hence we need to prove that $Q(b) > 0$ for any $b > 0$. After some tedious algebra, we can verify that all $\delta_i$s are positive. Thus $Q(b) > 0$ for any $b > 0$. This, in turn, proves that $Q(n) > 0$ and $\frac{\partial Var_t(\pi_t^{**})}{\partial n} > 0$. In addition, note that fixing $T \equiv \frac{1}{m} + \frac{1}{n}$, an increase in $n$ is the same as a decrease in $m$. Hence $\frac{\partial Var_t(\pi_t^{**})}{\partial m} < 0$.

*Proof of Proposition 6*

We only analyze the inflation volatility $Var_t(\pi_t^{**})$, as from Proposition 4 the results regarding the output $Var_t(x_t^{**})$ are the same. The third term in Equation (34) represents the fundamental volatility stemming from the innovations in the central bank's future inflation incentive $\{\nu_{t+1}, \nu_{t+2}\}$, defined below:

$$Var_t^F(\pi_t^{**}) \equiv \frac{\left[W_{\bar{s}_{t+1}}(m,n) + \rho_k W_{\bar{s}_{t+2}}(m,n)\right]^2 + \left[W_{s_{t+2}}(m,n)\right]^2}{q}. \tag{A.30}$$

The fourth term in Equation (34) is the non-fundamental volatility stemming from the noises in firms' signals, i.e., $\{\eta_{t+1}, \eta_{t+2}\}$, defined below:

$$Var_t^{NF}(\pi_t^{**}) \equiv \frac{\left[W_{\bar{s}_{t+1}}(m,n)\right]^2 + \left[W_{s_{t+2}}(m,n)\right]^2}{m}. \tag{A.31}$$

One can verify parts (i) and (ii) of the proposition, i.e., $\frac{\partial Var_t^F(\pi_t^{**})}{\partial n}, \frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial n}, \frac{\partial Var_t^F(\pi_t^{**})}{\partial m} > 0$ by directly computing the derivatives.

For the sign of $\frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial m}$ in part (iii) of the proposition, we can show that, after some tedious algebra,

$$\frac{\partial Var_t^{NF}\left(\pi_t^{**}\right)}{\partial m} = \frac{n^2 H\left(n\right)}{\left(1-\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\right)^2 \left[\left(mn+nq+mq\right)^2 + mnq\rho_k\left(\left(m+n\right)\rho_k - \frac{\alpha\beta}{\alpha+\lambda^2}m\right)\right]^3}. \quad \text{(A.32)}$$

It can be verified that the denominator of $\frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial m}$ is positive, so the sign of $\frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial m}$ is determined by $H\left(n\right)$, which is a fourth-order polynomial of $n$,

$$H\left(n\right) = \mu_1 n^4 + \mu_2 n^3 + \mu_3 n^2 + \mu_4 n^3 + \mu_5, \quad \text{(A.33)}$$

where the expressions of the coefficients $\{\mu_i\}_{i=1}^5$ are available upon request. We verify that $\mu_3 < 0$, $\mu_4 < 0$, $\mu_5 < 0$, and the signs of $\mu_1$ and $\mu_2$ are ambiguous. As a result, there can be the following four possible scenarios of the signs of $\{\mu_i\}_{i=1}^5$:

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\mu_5$ |
|---|---|---|---|---|
| + | + | − | − | − |
| + | − | − | − | − |
| − | + | − | − | − |
| − | − | − | − | − |

,

where "+" means positive and "−" means negative. In the first three cases, notice that for the polynomial $H\left(n\right)$, there is one sign change in its coefficients. Therefore, by Descartes's rule of signs, the polynomial $H\left(n\right)$ has a unique positive root. Denote this unique root as $\hat{n}'$, where $H\left(\hat{n}'\right) = 0$. For $n < \hat{n}'$, $H\left(n\right) < 0$ since at $n = 0$, $H\left(0\right) = \mu_5 < 0$ whereas for $n > \hat{n}'$, $H\left(n\right) > 0$. This, in turn, implies that $\frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial m} < 0$ if and only if $n < \hat{n}'$. In the last case, all $\mu_i$s are negative, so $H\left(n\right) < 0$ for all $n > 0$. In this case, $\frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial m} < 0$. Without loss of generality, define $\hat{n}' = \infty$ in this case. In sum, we have shown that there exists a unique $\hat{n}'$ such that $\frac{\partial Var_t^{NF}(\pi_t^{**})}{\partial m} < 0$ if and only if $n < \hat{n}'$.

*Proof of Proposition 7*

Note that, in (34), the first two terms in the inflation volatility $var\left(\pi_t^{**}\right)$ are both strictly increasing in $\alpha$. In addition, the weight $\alpha$ affects the last two terms of $var\left(\pi_t^{**}\right)$ only through affecting $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$ (i.e., the sensitivity effect) in (31) and (32), respectively. It is straightforward to verify that both $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$ are strictly in $\alpha$. Hence $var\left(\pi_t^{**}\right)$ is strictly increasing in $\alpha$.

*Proof of Proposition 8*

Collecting the first-order condition (45) and matching the coefficients in the equilibrium unemployment rate in Equation (43) with firms' conjecture in Equation (38) yields the following set of equations:

$$\mu = -\frac{1}{b}\left(1 + \frac{a\mu}{1 - a\mu\chi}\right)\frac{a}{1 - a\mu\chi}, \tag{A.34}$$

$$\delta = -\frac{a\mu}{1 - a\mu\chi}, \tag{A.35}$$

$$\lambda = \frac{a}{1 - a\mu\chi}, \tag{A.36}$$

$$\chi \equiv \frac{\delta\tau_\varepsilon}{\delta^2\tau_\varepsilon + \lambda^2\tau_k}. \tag{A.37}$$

Note first that $\delta = 0$ or $\mu = 0$ cannot be an equilibrium. To see this, assume by contradiction that there exists an equilibrium of $\delta = 0$. Hence $\mu = 0$ from Equation (A.35). In addition, $\chi = 0$ from Equation (A.37). Plugging $\mu = 0$ and $\chi = 0$ into Equation (A.34) gives its left-hand side as 0 but the right-hand side as $-\frac{a}{b}$, which implies that $\mu = 0$ does not solve the equation. This is a contradiction.

To determine the equilibrium, substituting Equation (A.35) into Equation (A.36) gives

$$\lambda = -\frac{\delta}{\mu}. \tag{A.38}$$

Substituting Equation (A.35) and Equation (A.36) into Equation (A.34) and using Equation (A.38) yields

$$\mu^2 = \frac{1}{b}(1-\delta)\,\delta. \tag{A.39}$$

This implies that $\delta \in (0,1)$ since $\mu^2 > 0$. Substituting Equation (A.38) and Equation (A.39) into Equation (A.37) yields

$$\chi = \frac{(1-\delta)\,\tau_\varepsilon}{(1-\delta)\,\delta\tau_\varepsilon + b\tau_k}. \tag{A.40}$$

Substituting Equation (A.40) into Equation (A.35) gives

$$\delta = -\frac{a\mu b\tau_k}{(1-\delta)\,\delta\tau_\varepsilon + b\tau_k}. \tag{A.41}$$

Since $\delta \in (0,1)$, the denominator of the right-hand side of Equation (A.41) must be positive, i.e., $(1-\delta)\,\delta\tau_\varepsilon + b\tau_k > 0$. Hence $\mu < 0$. Using Equation (A.39), we obtain

$$\mu = -\sqrt{\frac{1}{b}(1-\delta)\,\delta}. \tag{A.42}$$

Substituting Equation (A.42) into Equation (A.41) yields

$$\delta = \frac{a\sqrt{b\,(1-\delta)\,\delta}\,\tau_k}{(1-\delta)\,\delta\tau_\varepsilon + b\tau_k}. \tag{A.43}$$

The equilibrium value of $\delta \in (0,1)$ is determined by Equation (A.43). Using the implicit function theorem, one can show that $\delta$ is strictly increasing in $a$. Substituting (A.42) into Equation (A.38) gives

$$\lambda = \sqrt{\frac{b\delta}{1-\delta}}. \tag{A.44}$$

Since $\delta \in (0,1)$, $\lambda > 0$.

*Proof of Proposition 9*

This proof complements the steps given in the main text. First, we prove that at the steady state, in each period $t$, a fraction $\frac{1}{1+\theta}$ of the firms can reset the price whereas the remaining firms are restricted from resetting the price. Denote as $\Theta_t$ the fraction of the firms in period $t$ that can reset the price. Hence

$$\Theta_{t+1} = 1 - \Theta_t + \Theta_t (1 - \theta), \tag{A.45}$$

since the fraction $1 - \Theta_t$ of the firms that are restricted from resetting the price in period $t$ gains full price-resetting flexibility in period $t+1$ and the fraction $\Theta_t$ of the firms that reset the price in period $t$ can reset the price in period $t+1$ with probability $1-\theta$. In addition, the initial condition for iterating $\Theta_t$ is that $\Theta_0 = 1$ (i.e., all firms can reset the price at the beginning). Solving the difference, Equation (A.45) yields

$$\Theta_t = \frac{1}{1+\theta} - \frac{(-\theta)^{t+1}}{1+\theta}. \tag{A.46}$$

At the steady state, $\lim_{t\to\infty} \Theta_t = \frac{1}{1+\theta}$.

Next, we briefly explain that when $\frac{(\alpha-\lambda\kappa)\beta'}{\alpha+\lambda^2} > 0$, the effect of the informational properties $(m, n)$ on the volatilities of inflation and output is qualitatively the same as that in the main analysis. First, note that the hierarchy of the higher-order beliefs $\bar{E}_t^l [k_{t+l}]$ remains the same under the modified Phillips curve, i.e.,

$$\bar{E}_t^l [k_{t+l}] = \bar{k} + \rho_k^{l-1} \left\{ [1 - w(l)] \bar{E}_t \left[k_{t+1} - \bar{k} | s_{t+1}^i, k_t\right] + w(l) \frac{\bar{s}_{t+2} - \bar{k}}{\rho_k} \right\}, \tag{A.47}$$

where $\bar{E}_t \left[k_{t+1} - \bar{k}|s_{t+1}^i, k_t\right] = \frac{q}{q+\frac{mn}{m+n}} \rho_k \left(k_t - \bar{k}\right) + \frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}} \left(\bar{s}_{t+1} - \bar{k}\right)$. Second, substituting the expressions for $\bar{E}_t^l [k_{t+l}]$ into (57) yields

$$\pi_t^{**} = \frac{\alpha\lambda}{\alpha+\lambda^2} k_t + \sum_{l=1}^{\infty} \left( \frac{(\alpha - \lambda\kappa)\beta'}{\alpha+\lambda^2} \right)^{l-1} \frac{\alpha\beta'}{\alpha+\lambda^2} \frac{\alpha(\lambda+\kappa)}{\alpha+\lambda^2} \bar{k}$$

$$+\left\{\sum_{l=1}^{\infty}\left(\frac{(\alpha-\lambda\kappa)\,\beta'\rho_k}{\alpha+\lambda^2}\right)^{l-1}\frac{\alpha\beta'\rho_k}{\alpha+\lambda^2}\frac{\alpha\,(\lambda+\kappa)}{\alpha+\lambda^2}\right\}\left(k_t-\bar{k}\right)$$
$$+\left\{\sum_{l=1}^{\infty}\left(\frac{(\alpha-\lambda\kappa)\,\beta'\rho_k}{\alpha+\lambda^2}\right)^{l-1}\frac{\alpha\beta'}{\alpha+\lambda^2}\frac{\alpha\,(\lambda+\kappa)}{\alpha+\lambda^2}\right.$$
$$\left.\times\left[\frac{\bar{E}_t^l\left[k_{t+l}-\bar{k}\right]}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)\right]\right\}, \tag{A.48}$$

where

$$\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)=\left[1-w\left(l\right)\right]\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\nu_{t+1}+\eta_{t+1}\right)$$
$$+\,w\left(l\right)\left(\frac{\nu_{t+2}+\rho_k\nu_{t+1}+\eta_{t+2}}{\rho_k}\right). \tag{A.49}$$

Define

$$W'_{\bar{s}_{t+1}}=\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\frac{\alpha\beta'}{\alpha+\lambda^2}\frac{\alpha\,(\lambda+\kappa)}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{(\alpha-\lambda\kappa)\,\beta'\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left[1-w\left(l\right)\right], \tag{A.50}$$

$$W'_{\bar{s}_{t+2}}=\frac{\alpha\beta'}{\alpha+\lambda^2}\frac{\alpha\,(\lambda+\kappa)}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{(\alpha-\lambda\kappa)\,\beta'\rho_k}{\alpha+\lambda^2}\right)^{l-1}\frac{w\left(l\right)}{\rho_k}. \tag{A.51}$$

Dropping the terms that are independent of $(m, n)$, the inflation volatility $Var\left(\pi_t^{**}\right)$ can be expressed as

$$\frac{\left[W_{\bar{s}_{t+1}}+\rho_k W_{\bar{s}_{t+2}}\right]^2+W_{\bar{s}_{t+2}}^2}{q}+\frac{W_{\bar{s}_{t+1}}^2+W_{\bar{s}_{t+2}}^2}{m}. \tag{A.52}$$

Note that (A.52) is the same as the expression (34) for the inflation volatility in the main analysis, except that under the modified Phillips curve, the discounting factor in $\left\{W'_{\bar{s}_{t+1}}, W'_{\bar{s}_{t+2}}\right\}$ is $\frac{(\alpha-\lambda\kappa)\beta'\rho_k}{\alpha+\lambda^2}$ instead of $\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}$. However, since $\frac{(\alpha-\lambda\kappa)\beta'\rho_k}{\alpha+\lambda^2}\in(0,1)$, the derivations in the proof of Proposition 4 for the effect of the informational properties $(m, n)$ on the volatilities of inflation and output apply equally. Accordingly, the comparative statics results remain valid.

## References


Angeletos, G., and Z. Huo. 2021. "Myopia and Anchoring." *American Economic Review* 111 (4): 1166–1200.

Angeletos, G., and J. La'O. 2013. "Sentiments." *Econometrica* 81 (2): 739–79.

———. 2020. "Optimal Monetary Policy with Informational Frictions." *Journal of Political Economy* 128 (3): 1027–64.

Angeletos, G., and C. Lian. 2016. "Incomplete Information in Macroeconomics: Accommodating Frictions in Coordination." In *Handbook of Macroeconomics*, Vol. 2, ed. J. B. Taylor and H. Uhlig, 1065–1240. Elsevier.

———. 2018. "Forward Guidance without Common Knowledge." *American Economic Review* 108 (9): 2477–2512.

Angeletos, G., and A. Pavan. 2004. "Transparency of Information and Coordination in Economies with Investment Complementarities." *American Economic Review* 94 (2): 91–98.

———. 2007. "Efficient Use of Information and Social Value of Information." *Econometrica* 75 (4): 1103–42.

Ascari, G., and A. Sbordone. 2014. "The Macroeconomics of Trend Inflation." *Journal of Economic Literature* 52 (3): 679–739.

Barro, R., and D. Gordon. 1983a. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91 (4): 589–610.

———. 1983b. "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics* 12 (1): 101–21.

Baxter, M., and R. King. 1991. "Productive Externalities and Business Cycles." Discussion Paper No. 53, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis.

Benhabib, J., P. Wang, and Y. Wen. 2015. "Sentiments and Aggregate Demand Fluctuations." *Econometrica* 83 (2): 549–85.

Brunner, K. 1981. "The Art of Central Banking." Working Paper No. GPB 81-6, Center for Research in Government Policy and Business, University of Rochester.

Calvo, G. A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–98.

Clarida, R., J. Galí, and M. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (4): 1661–1707.

Cooper, R., and A. John. 1988. "Coordinating Coordination Failures in Keynesian Models." *Quarterly Journal of Economics* 103 (3): 441–63.

Cukierman, A., and A. Meltzer. 1986. "A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information." *Econometrica* 54 (5): 1099–1128.

Dotsey, M., and R. King. 1986. "Informational Implications of Interest Rate Rules." *American Economic Review* 76 (1): 33–42.

Feltham, G. 1973. *Information Evaluation.* Studies in Accounting Research No. 5. Sarasota, FL: American Accounting Association.

Galí, J. 2008. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework.* Princeton, NJ: Princeton University Press.

Geraats, P. 2002. "Central Bank Transparency." *Economic Journal* 112 (483): 532–65.

Goodfriend, M. 1986. "Monetary Mystique: Secrecy and Central Banking." *Journal of Monetary Economics* 17 (1): 63–92.

Goodfriend, M., and R. King. 2012. "The Great Inflation Drift." In *The Great Inflation: The Rebirth of Modern Central Banking*, ed. M. D. Bordo and A. Orphanides, 181–209. University of Chicago Press.

Hansen, L., and T. Sargent. 2007. "Recursive Robust Estimation and Control without Commitment." *Journal of Economic Theory* 136 (1): 1–27.

Hellwig, C., and L. Veldkamp. 2009. "Knowing What Others Know: Coordination Motives in Information Acquisition." *Review of Economic Studies* 76 (1): 223–51.

Holthausen, R., and R. Verrecchia. 1990. "The Effect of Informedness and Consensus on Price and Volume Behavior." *Accounting Review* 65 (1): 191–08.

Ijiri, Y., and R. Jaedicke. 1966. "Reliability and Objectivity of Accounting Measurements." *Accounting Review* 41 (3): 474–83.

King, R. 1982. "Monetary Policy and the Information Content of Prices." *Journal of Political Economy* 90 (2): 247–79.

———. 1983. "Interest Rates, Aggregate Information, and Monetary Policy." *Journal of Monetary Economics* 12 (2): 199–234.

King, R., and A. Wolman, eds. 2022. *Essays in Honor of Marvin Goodfriend: Economist and Central Banker*. Federal Reserve Bank of Richmond.

Kiyotaki, N., and R. Wright. 1989. "On Money as a Medium of Exchange." *Journal of Political Economy* 97 (4): 927–54.

Kydland, F., and E. Prescott. 1977. "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy* 85 (3): 473–91.

Liang, P., and G. Zhang. 2019. "On the Social Value of Accounting Objectivity in Financial Stability." *Accounting Review* 94 (1): 229–48.

Mailath, J., and L. Samuelson. 2006. *Repeated Games and Reputations.* Oxford: Oxford University Press.

McCallum, B. 1990. "Could a Monetary Base Rule Have Prevented the Great Depression?" *Journal of Monetary Economics* 26 (1): 3–26.

Mock, M. 1976. *Measurement and Accounting Information Criteria.* Studies in Accounting Research No. 13. Sarasota, FL: American Accounting Association.

Morris, S., and H. Shin. 2002. "Social Value of Public Information." *American Economic Review* 92 (5): 1521–34.

———. 2005. "Central Bank Transparency and the Signal Value of Prices." *Brookings Papers on Economic Activity* 2: 1–66.

Myatt, D., and C. Wallace. 2012. "Endogenous Information Acquisition in Coordination Games." *Review of Economic Studies* 79 (1): 340–74.

Paciello, L., and M. Wiederholt. 2014. "Exogenous Information, Endogenous Information, and Optimal Monetary Policy." *Review of Economic Studies* 81 (1): 356–88.

Phelps, E. 1983. "The Trouble with 'Rational Expectations' and the Problem of Inflation Stabilization." In *Individual Forecasting and Aggregate Outcomes: 'Rational Expectations' Examined*, ed. R. Frydman and E. S. Phelps, 31–45. Cambridge: Cambridge University Press.

Rogoff, K. 1985. "The Optimal Degree of Commitment to an Intermediate Monetary Target." *Quarterly Journal of Economics* 100 (4): 1169–89.

Rudebusch, G., and J. Williams. 2008. "Revealing the Secrets of the Temple: The Value of Publishing Interest Rate Projections." In

*Asset Prices and Monetary Policy*, ed. J. Y. Campbell. University of Chicago Press.

Sims, C. 2003. "Implications of Rational Inattention." *Journal of Monetary Economics* 50 (3): 665–90.

Stein, J., and A. Sunderam. 2018. "The Fed, the Bond Market, and Gradualism in Monetary Policy." *Journal of Finance* 73 (3): 1015–60.

Townsend, R. 1983. "Forecasting the Forecasts of Others." *Journal of Political Economy* 91 (4): 546–88.

Winkler, B. 2002. "Which Kind of Transparency? On the Need for Effective Communication in Monetary Policy-making." *Ifo Studies* 48 (3): 401–27.

Woodford, M. 2001. "Imperfect Common Knowledge and the Effects of Monetary Policy." NBER Working Paper No. 8673.

———. 2008. "How Important is Money in the Conduct of Monetary Policy?" *Journal of Money, Credit and Banking* 40 (8): 1561–98.

Yun, T. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37 (2): 345–70.