Online Appendices to "Exchange Rate Shocks and Inflation Co-movement in the Euro Area"

Danilo Leiva-Leon,[a] Jaime Martínez-Martín,[b] and Eva Ortega[a]
[a]Banco de España
[b]European Central Bank

Appendix A

A.1 Estimation of TVP Factor Model with Exogenous Information

The proposed estimation algorithm relies on Bayesian methods; in particular, we use the Gibbs sampler to approximate the posterior distribution of parameters and latent variables involved in the time-varying parameter factor model with exogenous information (TVP-DFX). Let the vectors of observed variables be defined as $\widetilde{\pi}_T = \{\pi_{1,t}, \ldots, \pi_{n,t}\}_{t=1}^{T}$, $\widetilde{x}_T = \{\epsilon_t^{Exo\_ER}\}_{t=1}^{T}$, and the vectors of latent variables as $\widetilde{f}_T = \{f_t\}_{t=1}^{T}$, $\widetilde{\lambda}_T = \{\lambda_t\}_{t=1}^{T}$, $\widetilde{\phi}_T = \{\phi_t\}_{t=1}^{T}$, and $\widetilde{\gamma}_T = \{\widetilde{\gamma}_{1,T}, \ldots, \widetilde{\gamma}_{i,T}, \ldots, \widetilde{\gamma}_{n,T}\}$, where $\widetilde{\gamma}_{i,T} = \{\gamma_{i,t}\}_{t=1}^{T}$, for $i = 1, \ldots, n$. The parameters of the model, which consist of the variances associated with the different innovation processes, are given by $\Sigma = diag(\sigma_1^2, \ldots, \sigma_n^2)$, $\Omega = \{\nu_1^2, \ldots, \nu_n^2\}$, $\Pi = diag(\nu_\lambda^2, \nu_\phi^2)$, and can be collected in $\Theta = \{\Sigma, \Omega, \Pi\}$ to simplify notation. The algorithm consists of the following steps:

- **Step 1:** Sample $\widetilde{f}_T$ from $P(\widetilde{f}_T | \widetilde{\pi}_T, \widetilde{x}_T, \widetilde{\lambda}_T, \widetilde{\phi}_T, \widetilde{\gamma}_T, \Theta)$.

We cast the proposed factor model in state-space representation, with measurement equation given by

$$\begin{bmatrix} \pi_{1,t} \\ \vdots \\ \pi_{n,t} \end{bmatrix} = \begin{bmatrix} \gamma_{1,t} \\ \vdots \\ \gamma_{n,t} \end{bmatrix} f_t + \begin{bmatrix} u_{1,t} \\ \vdots \\ u_{n,t} \end{bmatrix}, \tag{A.1}$$

and transition equation defined as

$$f_t = \mu_t + \phi_t f_{t-1} + \omega_t, \tag{A.2}$$

where $\mu_t = \lambda_t \epsilon_t^{Exo\_ER}$ and, similarly to other parameters of the state-space model, are observed in this step of the algorithm. The innovations are assumed to be Gaussian, $(u_{1,t}, \ldots, u_{n,t})' \sim N(0, \Sigma)$, and $\omega_t \sim N(0, 1)$. Notice that the variance ω_t is set to one; this restriction is assumed for identification of the factor model. Conditional on the time-varying parameters being observed, the Carter and Kohn (1994) simulation smoother is applied to generate inferences of the latent factor, f_t.

- **Step 2:** Sample $\widetilde{\gamma}_T$ from $P(\widetilde{\gamma}_T | \widetilde{\pi}_T, \widetilde{f}_T, \Omega, \Sigma)$.

Given that Σ is a diagonal matrix, we sample the time-varying factor loadings associated with each observable independently from each other by employing the following state-space representation:

$$\begin{aligned} \pi_{i,t} &= \gamma_{i,t} \;\; f_t \;\; + u_{i,t}, \\ \gamma_{i,t} &= \gamma_{i,t-1} + \vartheta_{i,t}, \end{aligned}$$

where $u_{i,t} \sim N(0, [\Sigma_{ii}])$ and $\vartheta_{i,t} \sim N(0, \nu_i^2)$, for $i = 1, \ldots, n$. Conditional on the factor, f_t, being observed, the Carter and Kohn (1994) simulation smoother is applied to generate inferences of the factor loadings, $\gamma_{i,t}$.

- **Step 3:** Sample Ω from $P(\Omega | \widetilde{\gamma}_T)$.

We sample the elements of $\Omega = \{\nu_1^2, \ldots, \nu_n^2\}$ conditional on the dynamics of the time-varying factor loadings by relying on a prior inverse-gamma distribution, $IG(\underline{\eta}, \underline{v})$, with $\underline{\eta} = \kappa \times T$, and $\underline{v} = 0.01 \times (\underline{\eta} - 1)$. The coefficient κ measures the degree of uncertainty about the prior belief of the innovations variance of the factor loadings. The larger (smaller) the κ, the smaller (larger) the uncertainty about the prior belief. If there is a relatively high (low) degree of underlying co-movement, a factor model would be more (less) suitable for the data, and the uncertainty about the dynamics of the factor loadings would be smaller (larger). Therefore, we set $\kappa = 0.1 \times std^{-1}$, where *std* measures the median, cross-sectional and over time, of the squared differences of inflation between two countries, which provides a simple measures of overall co-movement in the data. Accordingly, draws are sampled from independent posterior distributions

$$\nu_i^2 \sim IG(\bar{\eta}, \bar{v}),$$

with $\bar{\eta} = \underline{\eta} + T$, and $\bar{v} = \underline{v} + (\gamma_{i,t} - \gamma_{i,t-1})'(\gamma_{i,t} - \gamma_{i,t-1})$, for $i = 1, \ldots, n$.

- **Step 4:** Sample Σ from $P(\Sigma|\widetilde{\pi}_T, \widetilde{f}_T, \widetilde{\gamma}_T)$.

We sample the elements of $\Sigma = diag(\sigma_1^2, \ldots, \sigma_n^2)$ conditional on the observed data, factor, and time-varying factor loadings by relying on a prior inverse-gamma distribution, $IG(\underline{\eta}, \underline{v})$. Hence, draws are sampled from independent posterior distributions

$$\sigma_i^2 \sim IG(\bar{\eta}, \bar{v}),$$

with $\bar{\eta} = \underline{\eta} + T$, and $\bar{v} = \underline{v} + (\pi_{i,t} - \gamma_{i,t} f_t)'(\pi_{i,t} - \gamma_{i,t} f_t)$, for $i = 1, \ldots, n$.

- **Step 5:** Sample $\widetilde{\lambda}_T, \widetilde{\phi}_T$ from $P(\widetilde{\lambda}_T, \widetilde{\phi}_T|\widetilde{f}_T, \widetilde{x}_T, \Pi)$.

We sample jointly the time-varying coefficients, $\widetilde{\lambda}_T, \widetilde{\phi}_T$, by using the following state-space representation:

$$f_t = \begin{bmatrix} f_{t-1} & \epsilon_t^{Exo\_ER} \end{bmatrix} \begin{bmatrix} \phi_t \\ \lambda_t \end{bmatrix} + \omega_t,$$

$$\begin{bmatrix} \phi_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} \phi_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} \vartheta_{\phi,t} \\ \vartheta_{\lambda,t} \end{bmatrix},$$

where $\omega_t \sim N(0,1)$ and $(\vartheta_{\phi,t}, \vartheta_{\lambda,t})' \sim N(0,\Pi)$. Conditional on the dynamics of the factor and the exogenous variable being observed, the Carter and Kohn (1994) simulation smoother is applied to generate inferences of the time-varying coefficients.

- **Step 6:** Sample Π from $P(\Pi|\widetilde{\lambda}_T, \widetilde{\phi}_T)$.

We sample the elements of $\Pi = diag(\nu_\lambda^2, \nu_\phi^2)$ conditional on the dynamics of the corresponding time-varying coefficients by relying on a prior inverse-Wishart distribution, $IW(\underline{\eta}, \underline{V})$, with $\underline{V} = I_2 \times \underline{v}$. Hence, draws are sampled from the posterior distribution

$$\Pi \sim IW(\bar{\eta}, \bar{V}),$$

with $\bar{\eta} = \underline{\eta} + T$, and $\bar{V} = \underline{V} + (\xi_t - \xi_{t-1})'(\xi_t - \xi_{t-1})$, where $\xi_t = (\phi_t, \lambda_t)'$.

To approximate the posterior distribution of both the parameters and latent variables involved in the model, each step of the algorithm is recursively repeated $M = 20{,}000$ times, discarding the first $m = 10{,}000$ iterations.

A.2 Shock-Dependent EPRT and Open-Economy Macroeconomic Theory

In this section of the appendix, we aim to shed some light on the link between our shock-dependent approach and a standard open-economy model in order to illustrate how and why pass-through depends on the underlying shocks moving the EUR/USD exchange rate. Theory suggests a number of ways in which exchange rate pass-through is shock dependent. However, a major concern in this type of analysis is to use economic theory to identify the shocks of interest with appropriate restrictions on variables' impulse responses. In addition, new open-economy macroeconomic models are well known to face another major challenge: they have a clear difficulty to combine a relatively important exchange rate pass-through (ERPT) at the border with low pass-through at the consumer level. The low pass-through problem has been mainly addressed by augmenting the standard model with a domestic distribution sector according to the intuition of Corsetti and Dedola (2005).

To overcome these two challenges, we impose several short-run sign restrictions—summarized in Table B.1—which are motivated by open-economy DSGE models. In particular, these restrictions are consistent with the two-country New Keynesian model described in de Walque et al. (2017), the Banque Nationale de Belgique model of the euro-area economy, which entails the standard open-economy characteristics. This DSGE model integrates two closed-economy models (for the euro area and the United States, based on Smets and Wouters 2007), through international trade in goods and assets, and it is rather rich in terms of features: sticky local-currency pricing, distribution sector, intermediate goods in the production function, and a demand elasticity increasing with the relative price. These features help to reduce the exchange rate pass-through to import price at the border and down the chain towards consumption price, both in the short run and in the long run.[1]

More specifically, our SVAR identification scheme is generally comparable and closely matched (up to some unrestricted responses) to that in the structural model of reference, de Walque et al. (2017).[2] Let us briefly describe the endogenous reactions to those five unanticipated shocks and discuss the impulse response functions from the euro-area point of view.

First, following an expansionary euro-area aggregate demand shock, the initial excess demand is absorbed by an increase in GDP and a gradual increase in prices, prompting a countercyclical monetary policy response by the central bank by raising the interest rate. As a consequence, the euro appreciates.[3] Firms face a persistent increase in demand for their products and increase their markup.[4] Therefore, import prices follow the appreciation but fall by less than they would have without the improvement in domestic conditions. Total consumer prices initially decrease in line with the drop in import prices, but after a while, the consumer price impulse response

[1]We only focus on understanding how ERPT depends on the underlying shocks moving the exchange rate, while pass-through could also vary due to structural factors. These include, but are not limited to, the non-linear response of prices to the size of exchange rate changes, the degree of participation in global value chains, firms' market power, and the choice of invoicing currency for international trade.

[2]The one restriction that does not match between the SVAR and the DSGE model is the impact of the domestic supply shock on inflation: in the SVAR, a restriction is imposed at impact to set-identify the supply shock from the demand shock, while in the DSGE model the effect at impact is not different from zero. The opposite signs for demand versus supply shocks to prices only appear and become substantial after three to four quarters.

[3]The switching effect is negative on exports and positive on imports. Import prices increase due not only to the favorable movement in relative prices but also to the higher euro-area aggregate demand. Import prices decrease in line with the exchange rate appreciation.

[4]The higher weight of domestic versus imported goods (home bias) in the demand of most European economies ensures that cheaper imported inflation does not prevail over the inflationary effect of the positive demand shock.

turns positive, following the increase in prices of domestic goods and the bottoming-out of prices of imported goods. These sign restrictions have been widely used in the literature and have been shown to be consistent with theoretical models—for example, in Fry and Pagan (2011).

Second, after a positive supply shock, the excess supply is absorbed through the fall in domestic costs and prices.[5] Given the large home bias, total consumer prices decrease. The central bank reduces the policy rate, stimulating domestic aggregate demand and triggering nominal exchange rate depreciation. Import prices increase, consistent with the depreciation and increase in aggregate demand. Total consumer prices still gradually decrease, as they are dampened by the fall in prices of domestic goods. The EPRT to import prices is positive both at the border and at the retail level, while that for consumer prices is negative.

Third, following an unexpected relative monetary policy tightening in the euro area with respect to that of the U.S. Federal Reserve, the higher (real) interest rate induces an appreciation in the euro through the uncovered interest rate parity (UIP) condition and a decrease in euro-area aggregate demand for consumption and investment. Foreign firms exporting to the euro area react by reducing the prices of their goods invoiced in euros. Due to the nominal rigidities, the reduction is gradual and not as large as the appreciation in the euro, and firms adjust their markups temporarily. The consumer price index decreases consistently with the decrease in import prices and euro-area aggregate demand. However, it does so sluggishly because of the higher stickiness and higher share of domestically produced goods.

Fourth, in the case of an unexpected euro appreciation due to a purely exogenous change—a risk premium shock based on neither real activity fundamentals nor monetary policy—there are two main initial responses: (i) import prices at the border decrease following the nominal exchange rate appreciation, and (ii) agents substitute Eurodollar-denominated assets for U.S.-denominated assets due to

[5]Consistent with earlier literature, such as Canova and de Nicolo (2003) and Forbes, Hjortsoe, and Nenova (2015), if the exchange rate movement is related to a supply shock, one would expect a negative correlation between euro-area growth and inflation.

the UIP condition. Aggregate import prices adjust only gradually to the changes in the exchange rate because of price stickiness: individual firms adjust short-run markups. Import prices at the retail (i.e., consumer) level adapt to a lower extent than at the border. The prices of the domestic distribution services—which introduce a wedge between the import prices at the border and the retail level—evolve at an even slower pace. Import prices push total consumer prices downwards, in spite of them having a relatively small weight in the consumer price index. Prices of domestic goods, associated with a much larger weight in the index, also decrease due to the decrease in the price of intermediate foreign inputs and to the shift in domestic and foreign demand towards foreign goods. In this case, the central bank cuts the policy rate in response to the deflationary pressures in line with An and Wang (2012)[6] and the small negative effect on the euro-area GDP,[7] mitigating somewhat the appreciation in the euro. In fact, through the UIP condition, the lower domestic interest rate relative to the foreign interest rate counteracts the initial appreciation in the currency. Monetary policy is expected to tighten the interest rate.

Fifth, let us consider a scenario where the shock driving the euro exchange rate is a positive global demand shock. This would be associated with an increase in the euro activity and prices. Given that demand growth in the rest of the world would be higher, the corresponding countercyclical monetary policy response by tightening interest rates would probably be stronger than in the domestic economy, thus actually easing monetary policy in relative terms.

A.3 Robustness Checks

With the aim of assessing if our results reported in Section 2.1 are robust to different specifications, we summarize a series of extensions

[6] An alternative identification strategy relaxing the latter assumption is shown in the next section of this appendix, with broadly similar results.

[7] The higher monetary policy rate induces households and firms to reduce consumption and investment. At the same time, and for the same reasons that import prices increase, the export prices expressed in the currency of the destination market decrease. The implied expenditure-switching effect favors exports and reduces imports, improving the net trade. The net effect on the euro-area real economic activity is slightly positive.

and sensitivity checks. First, to alternatively identify the structural shocks with regards to the unexpected appreciation of the euro, we rely on imposing a different set of sign restrictions in some of the entries of the impact multiplier matrix. Second, we analyze any effect of changes in the SVAR dynamic properties such as model lag orders and timing of sign restrictions. Third, although the paper only analyzes the EUR/USD exchange rate, we have simulated an alternative SVAR model considering the nominal effective exchange rate of the euro (NEER-38) instead of the bilateral EUR/USD exchange rate. Fourth, we evaluate the differences across the shocks extracted from our baseline SVAR model and a version of the same specification, but subject to time-varying parameters (TVP-VAR). Fifth, we evaluate the role of oil prices under an SVAR-X model.

A.3.1 Alternative Identification Strategies

Let us now assume that an unexpected appreciation of the euro, $\epsilon_t^{Exo\_ER}$, would lead to declines in inflation, along with further appreciation of the euro and a rise in output (through confidence channels) and in the global demand. In the baseline scenario, monetary policy is expected to loose the interest rate in line with An and Wang (2012). However, an alternative identification strategy relaxing the latter assumption is imposed (i.e., with no assumption about whether the interest rate is unchanged or lowered). All these restrictions can be formalized as follows:

$$\begin{bmatrix} u_t^{GDP} \\ u_t^{INF} \\ u_t^{INT} \\ u_t^{FX} \\ u_t^{EA/US} \end{bmatrix} = \begin{bmatrix} + & + & - & + & + \\ - & + & - & - & + \\ - & + & + & * & * \\ - & * & * & + & * \\ + & + & - & + & - \end{bmatrix} \begin{bmatrix} \epsilon_t^{Dom\_Sup} \\ \epsilon_t^{Dom\_Dem} \\ \epsilon_t^{Mon\_Pol} \\ \epsilon_t^{Exo\_ER} \\ \epsilon_t^{Glo\_Dem} \end{bmatrix},$$

where an asterisk ($*$) in the impact multiplier matrix indicates that such a relation is left unrestricted. As shown in Figure B.1, the historical decomposition of shocks is little changed with respect to the baseline identification strategy (Figure 1 in the main paper).

A.3.2 Alternative SVAR Dynamics

As an additional set of robustness checks, we analyze any effect of changes in the SVAR specification, in terms of lag orders and timing of sign restrictions in the vein of Forbes, Hjortsoe, and Nenova (2018). The results in Table B.1 (columns 2–4) show no remarkable differences by changing the lag structure compared to our baseline results (lag of order 2). In addition, our results do not seem to be sensitive to imposing longer sign restrictions of two or four quarters (columns 5 and 6).

A.3.3 EUR/USD versus NEER

Despite a growing role of the euro in international trade, the importance of the U.S. dollar for domestic prices can be found in its special position as the most important invoicing currency in international trade. The share of the dollar as the invoicing currency is much higher compared to the actual exports to the United States (Goldberg and Tille 2009), and around 95 percent of extra-EU imports are invoiced in euro and U.S. dollars, which has a share in extra-EU imports of goods by country ranging from over 30 percent for Austria to 62 percent for Ireland (Ortega and Osbat 2020).

To provide some insights on how important this particular exchange rate, EUR/USD, is for the euro area, it is worthy to mention that up till 2006, the weight of the euro-area trading partner countries in the EER-19 indices provided by the European Central Bank has been led by the United States (21.5 percent on average). From then on, China took the lead with more than 23 percent of the total in the reference period 2013–15, followed by the United States and the United Kingdom (16.9 percent and 12.9 percent, respectively). Therefore, although it is important to highlight the predominance of the EUR-USD relationship, one could quibble about the effects of omitted euro-area trading partners. We simulate an alternative SVAR model considering the nominal effective exchange rate of the euro (NEER-38) instead of the bilateral EUR/USD exchange rate. Our results suggest that these omissions do not modify our findings.

A.3.4 Time-Varying Parameter SVAR

The dynamic properties of those series accounted for in our shock-dependent approach might not be constant over time. For this reason, we assess whether our main results are robust to a specification that allows both the estimated coefficients and the residuals covariance matrix to change over time. Let Y_t be an n-vector of time series satisfying

$$\mathbf{Y}_t = A_{0,t} + A_{1,t}\mathbf{Y}_{t-1} + \ldots + A_{p,t}\mathbf{Y}_{t-p} + \epsilon_t, \tag{A.3}$$

where ϵ_t is Gaussian white noise mean and time-varying covariance matrix Σ_t and $A_{j,t}$ are matrices of coefficients $(n \times n)$. For the law of motion of the VAR parameters, let $A_t = [A_{0,t}, A_{1,t}, \ldots, A_{p,t}]$ and $\theta_t = vec(A_t')$, where

$$\theta_t = \theta_{t-1} + \omega_t, \tag{A.4}$$

where ω_t is Gaussian white noise with zero mean and covariance Ω. In addition, let the covariance matrix be $\sum_t = F_t D_t F_t'$, where F_t is lower triangular and D_t is a diagonal matrix. Finally, the law of motion of the covariance matrix is defined as follows. First, let σ_t be the n-vector of the diagonal elements of $D_t^{1/2}$ and let $\phi_{i,t}$, with $i = 1, \ldots, n-1$, be the column vector formed by the non-zero and non-one elements of the $(i+1)$-th row F_t^{-1}. We assume that

$$\log \sigma_t = \log \sigma_{t-1} + \xi_t \tag{A.5}$$

$$\phi_{i,t} = \phi_{i,t-1} + \psi_{i,t}, \tag{A.6}$$

where ξ_t and $\psi_{i,t}$ are again Gaussian white noises with zero mean and covariance matrix Ξ and Ψ_i, respectively. Let us also assume that ξ_t, $\psi_{i,t}$, ω_t, and ϵ_t are mutually orthogonal at all leads and lags.

Finally, the estimation procedure is based on Bayesian Markov chain Monte Carlo (MCMC) methods (Gibbs sampler) in order to obtain the draws of the coefficients from the posterior distribution with the same identification strategy as previously mentioned. Let the vector ϕ_t be a vector containing all the $\phi_{i,t}$, $i = 1, \ldots, n-1$, and σ^T containing $\sigma_1, \sigma_2, \ldots, \sigma_T$. The posterior distribution is unknown, but not the conditional posteriors:

- **Step 1:** Sample σ^T from $p(\sigma^T|Y^T, \theta^T, \phi^T, \Omega, \Xi, \Psi)$.
- **Step 2:** Sample ϕ^T from $p(\phi^T|Y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$.
- **Step 3:** Sample θ^T from $p(\theta^T|Y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$.
- **Step 4:** Sample Ω from $p(\Omega|Y^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi)$.
- **Step 5:** Sample Ξ from $p(\Xi|Y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi)$.
- **Step 6:** Sample Ψ from $p(\Psi|Y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi)$.

We generate $M = 10{,}000$ iterations, and discard the first $m = 1{,}000$ iterations. Time-varying impulse response functions computed at each quarter do not significantly vary over the sample period, as is shown in Figure B.2. Also, historical decomposition of shocks suggests that there are no serious grounds for parameter instability to change our main results.

A.3.5 The Role of Oil Prices: SVAR-X

A final important related aspect is the link between oil prices and exchange rate developments. A relevant example on the causal relationship between oil prices and exchange rates is Akram (2009). His findings suggest that commodity prices increase significantly in response to reductions in real interest rates. Moreover, oil prices and prices of industrial raw materials tend to display overshooting behavior in response to such interest rate changes. His evidence also suggests that a weaker dollar leads to higher commodity prices. Yet, his SVAR restrictions apply to real exchange rate, being only able to investigate the impact of shocks on the real exchange rate. Therefore, it is not possible to examine pass-through from the nominal exchange rate to consumer prices within his model, but only the correlation between the real exchange rate and both the real interest rate and output.

Although very recent related literature point to oil prices as being generally endogenous with respect to (U.S.) exchange rate movements and monetary policy (Kilian and Zhou 2019), this section opts to be cautious about the source of the possible correlation between oil prices and the euro exchange rate. We acknowledge its influence, but rather use oil-price developments as an exogenous variable in the following empirical estimation of the structural shocks that drive the euro exchange rate over time. One of the main reasons is that, in such a setting, supply shocks driving the EUR/USD exchange rate and proxied by HICP inflation could be masking the effects of world oil prices.

Accordingly, to provide additional robustness tests to our empirical strategy, we estimate an alternative, endogenous multivariate model that uses quarterly information about the euro-area real GDP growth rate (GDP), euro-area HICP inflation (INF), relative short-term interest rates (INT) between the euro area and the United States, the EUR/USD nominal exchange rate (FX), and the relative euro-area activity share with respect to the United States (EA/US), as well as an additional exogenous component for world oil prices (OIL). Therefore, letting $\mathbf{Y}_t = [GDP_t, INF_t, INT_t, FX_t, EA/US_t]$ and $\mathbf{X}_t = [OIL_t]$, the estimated model is a structural vector autoregression with exogenous information SVAR-X(p, q)[8] given by

$$\mathbf{Y}_t = \Phi_0 + \sum_{p=1}^{P} \Phi_p \mathbf{Y}_{t-p} + \sum_{q=1}^{Q} \Phi_q \mathbf{X}_{t-q} + \mathbf{B}\boldsymbol{\epsilon}_t, \tag{A.7}$$

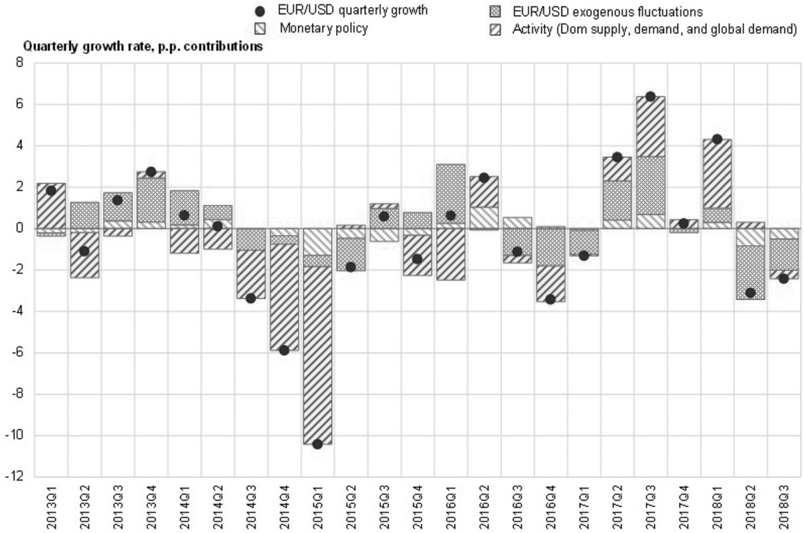
where $\boldsymbol{\epsilon}_t \sim N(0, I)$ are the structural innovations and $\mathbf{X}_t$ is assumed to be uncorrelated with $\boldsymbol{\epsilon}_t$ for all leads and lags. The reduced-form innovations, defined as $\mathbf{u}_t$, are related to the structural innovations through the impact multiplier matrix $\mathbf{B}$, that is, $\mathbf{u}_t = \mathbf{B}\boldsymbol{\epsilon}_t$. Figure B.3 reports the EUR/USD historical decomposition of shocks once oil prices are considered. As shown, there are few changes with respect to the EUR/USD historical decomposition obtained from the SVAR baseline model (Figure 1 in the main paper).

To summarize, all exogenous exchange rate shocks extracted from all three different structural model-based approaches (i.e., SVAR, TVP-SVAR, and SVAR-X) show little to no variation, since their statistical correlation is higher than 0.9 in all alternatives.

[8]In our empirical application, we let the number of lags of the endogenous variables $p = 2$ and that of the exogenous variable $q = 2$.

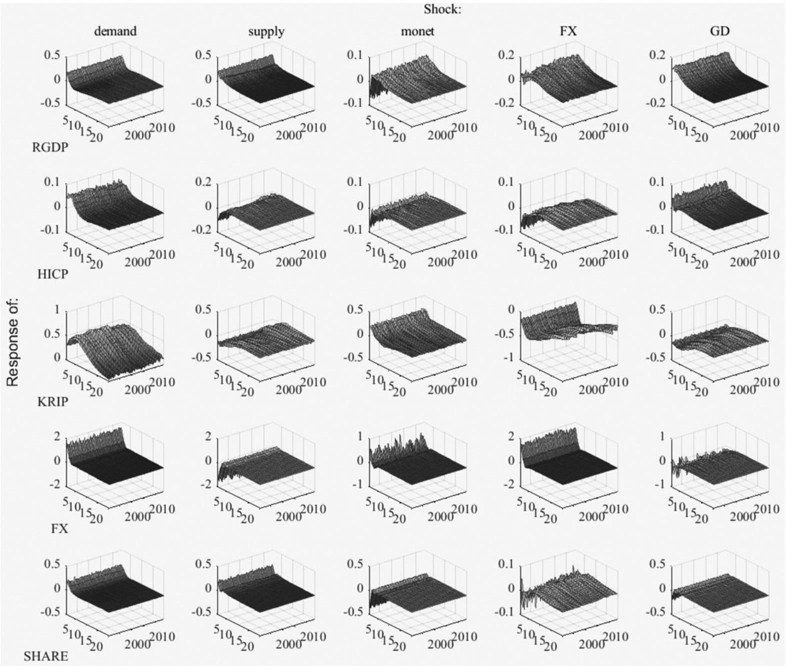
Appendix B. Figures and Table

Figure B.1. Historical Decomposition of Nominal Exchange Rate USD/EUR: Alternative



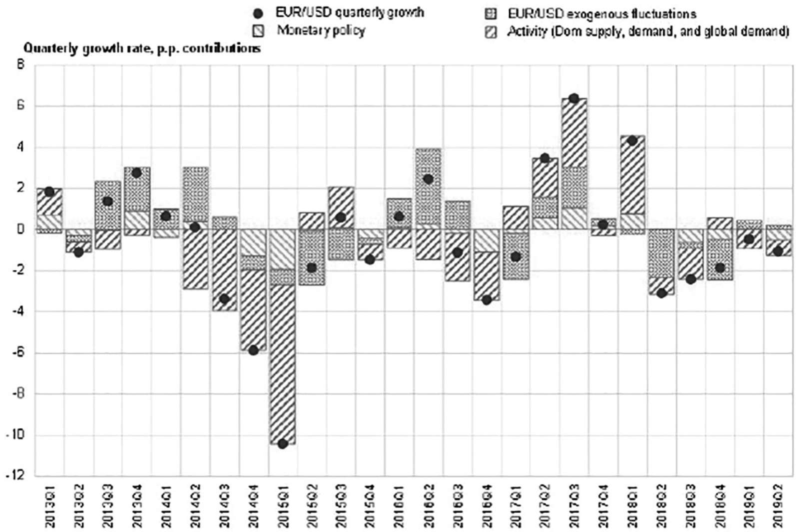
Note: Estimates based on a quarterly SVAR model of the USD/EUR exchange rate where shocks are identified via sign restrictions defined in Appendix A.

Figure B.2. Time-Varying Impulse Response Functions: TVP-SVAR Model with Sign Restrictions



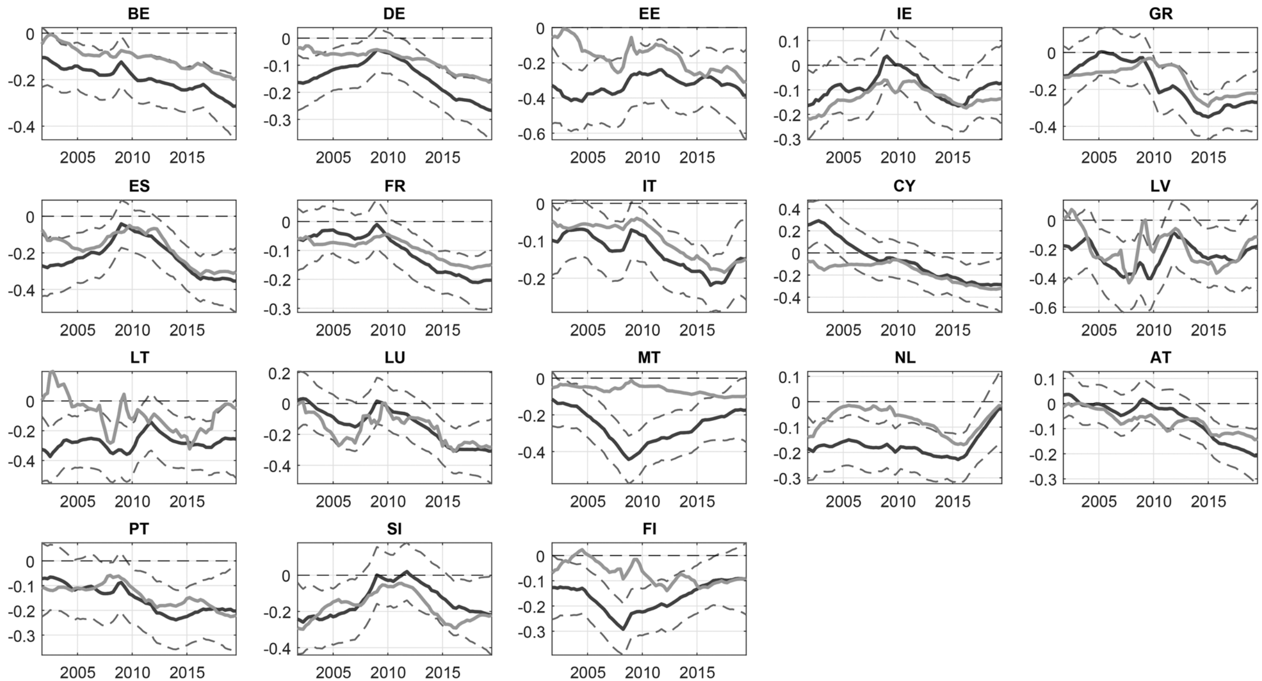
Note: Estimates based on a quarterly TVP-SVAR model of the USD/EUR exchange rate where shocks are identified via sign restrictions. RGDP refers to euro-area GDP growth, HICP refers to consumer prices inflation, KRIP refers to relative monetary policy rates for the euro area and the United States as of Krippner (2013), FX refers to the nominal USD/EUR exchange rate, and "share" refers to the relative share of activity growth between the euro area and the United States.

Figure B.3. Historical Decomposition of Nominal Exchange Rate EUR/USD: SVAR Model with Oil Prices as Exogenous Variable



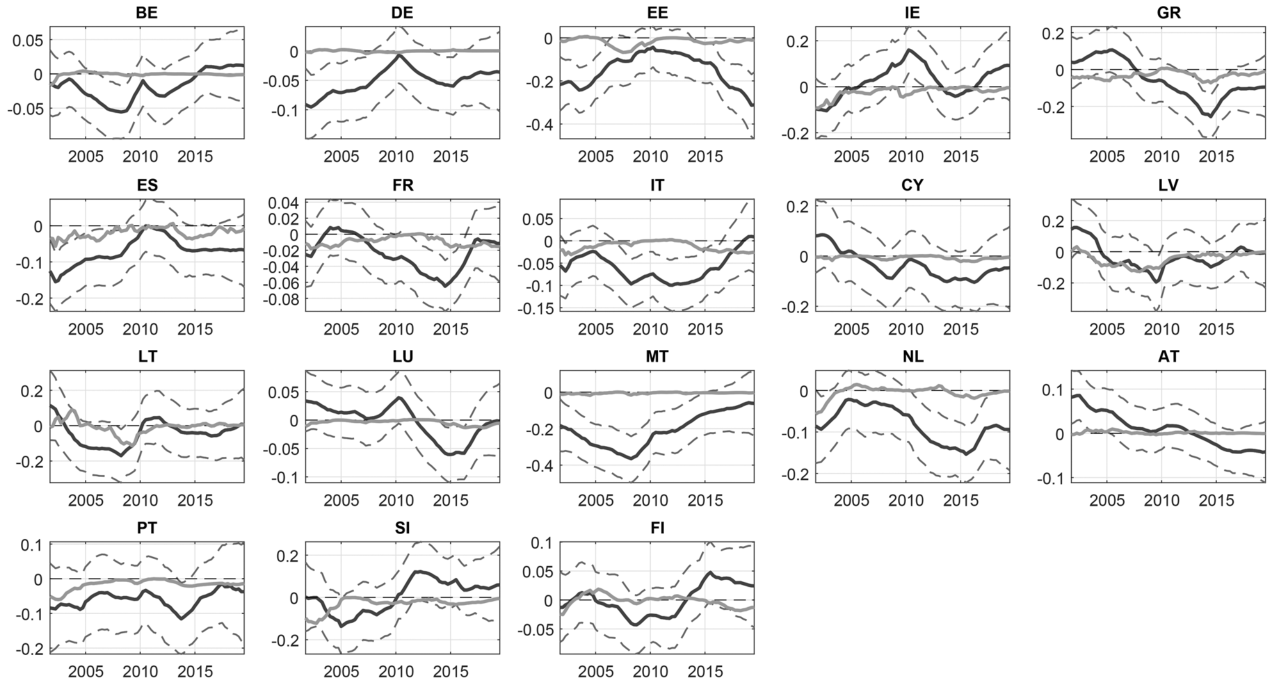
Note: Estimates based on a quarterly SVAR model with oil prices as exogenous variable (SVAR-X), as defined in Section A.3.5.

Figure B.4. Sensitivity of Country-Specific Headline Inflation to Exchange Rate Shocks Based on a Univariate Model ($\tilde{\beta}_{i,2,t}$)



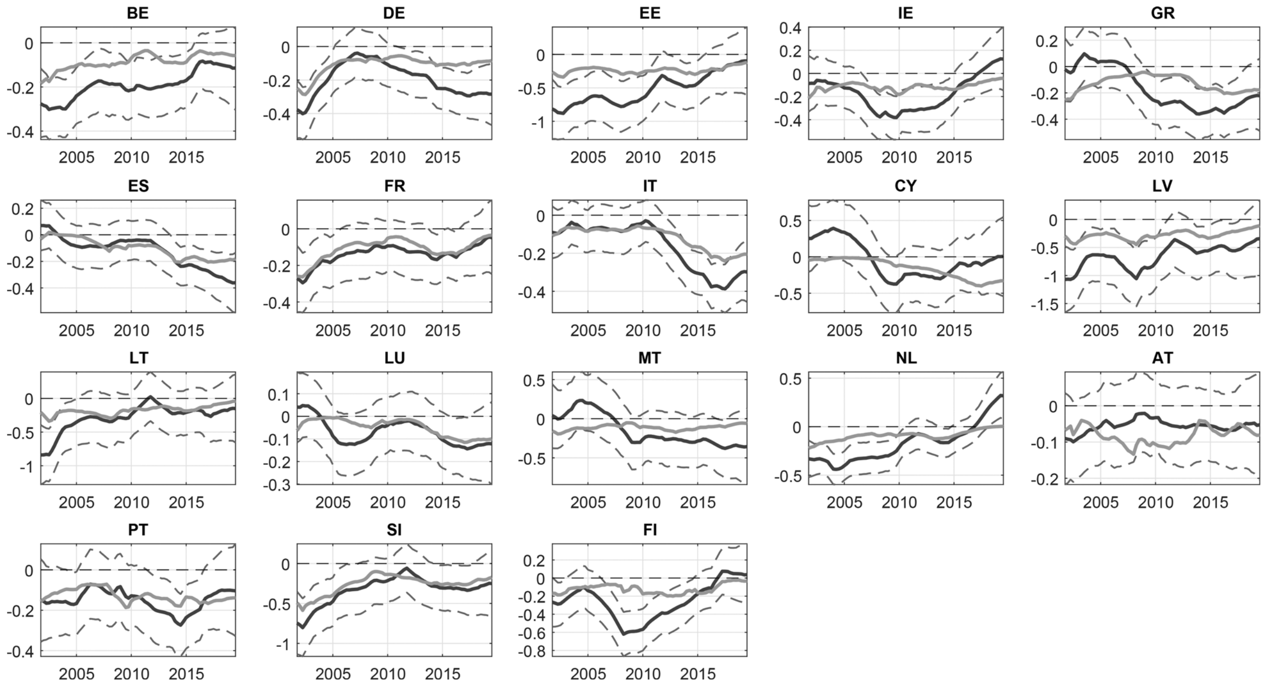
Note: Blue solid (red dashed) line makes reference to the median (16th and 84th percentile) of the posterior distribution estimates obtained with the univariate model. Green solid line reports the median estimate obtained with the multivariate model for comparison purposes.

Figure B.5. Sensitivity of Country-Specific Core Inflation to Exchange Rate Shocks Based on a Univariate Model ($\tilde{\beta}_{i,2,t}$)



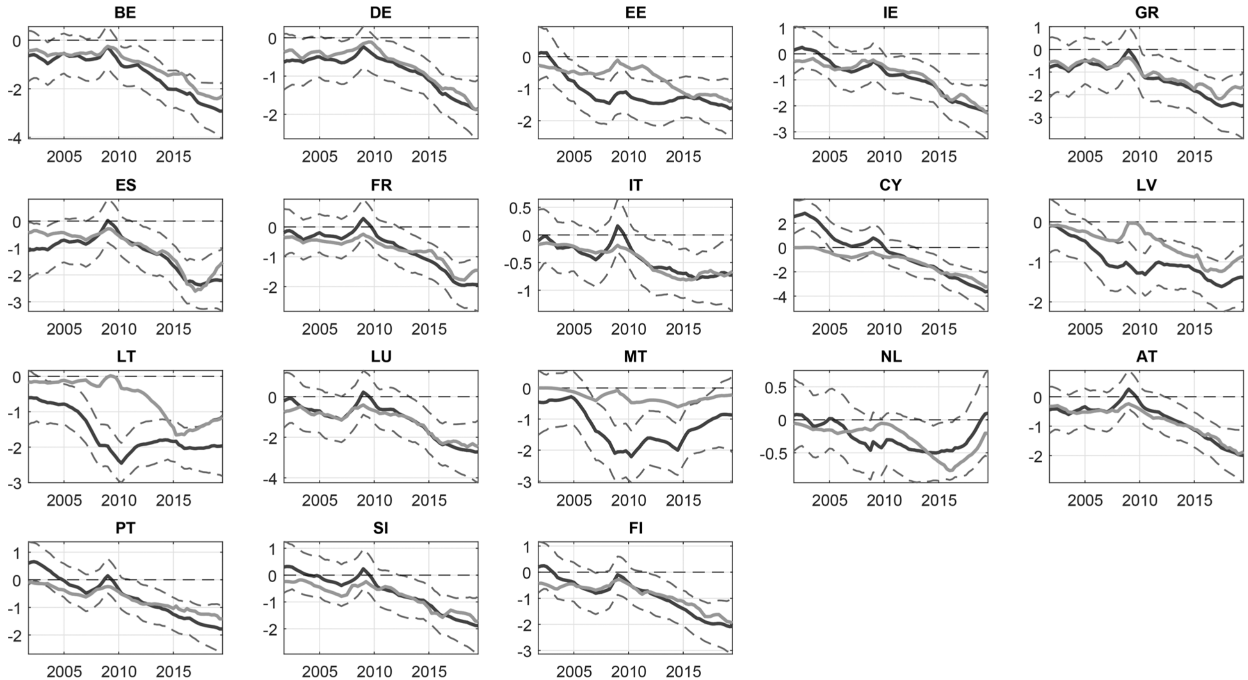
Note: Blue solid (red dashed) line makes reference to the median (16th and 84th percentile) of the posterior distribution estimates obtained with the univariate model. Green solid line reports the median estimate obtained with the multivariate model for comparison purposes.

Figure B.6. Sensitivity of Country-Specific Food-Related Inflation to Exchange Rate Shocks Based on a Univariate Model ($\tilde{\beta}_{i,2,t}$)



Note: Blue solid (red dashed) line makes reference to the median (16th and 84th percentile) of the posterior distribution estimates obtained with the univariate model. Green solid line reports the median estimate obtained with the multivariate model for comparison purposes.

Figure B.7. Sensitivity of Country-Specific Energy-Related Inflation to Exchange Rate Shocks Based on a Univariate Model ($\tilde{\beta}_{i,2,t}$)



Note: Blue solid (red dashed) line makes reference to the median (16th and 84th percentile) of the posterior distribution estimates obtained with the univariate model. Green solid line reports the median estimate obtained with the multivariate model for comparison purposes.

Table B.1. FEVD of the USD/EUR for Different Lag Orders and Sign Restriction Periods

	SVAR Estimated with:					
	Baseline (1)	**1 Lag (2)**	**3 Lags (3)**	**4 Lags (4)**	**2-per (5)**	**4-per (6)**
Domestic Demand	14%	17%	15%	17%	23%	24%
Domestic Supply	31%	29%	32%	31%	27%	30%
Rel. Monetary Policy	16%	15%	13%	13%	11%	14%
Exchange Rate	25%	24%	25%	23%	27%	21%
Global Demand	15%	15%	15%	16%	12%	11%

Note: Estimated using SVAR model described in Section 2.1. N-per refers to sign restrictions of N periods.

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