Risk Shocks and Monetary Policy in the New Normal*

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Risk shocks give rise to a tradeoff for monetary policy between inflation and output stabilization in the canonical New Keynesian model if they are large relative to the distance between the nominal interest rate and its lower bound. The tradeoff-inducing effects operate through expectational responses to the interaction between the perceived volatility of conventional level shocks and the available monetary policy space. At the same time, a given monetary policy stance becomes less effective. Optimal time-consistent monetary policy therefore calls for potentially sharp cuts in interest rates when risk is perceived to be elevated, even if this risk does not materialize in any actual disturbances to the economy. The new normal for monetary policy may be one in which policymakers should both constantly lean against a tendency for inflation expectations to anchor below target—operating the economy above potential in the absence of disturbances—whilst accepting that inflation will settle potentially materially below target, and respond nimbly to changes in public perceptions of economic risk.

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1. Introduction

During the Great Moderation, there was a general consensus that spells at the zero lower bound (ZLB) would be rare and short (see, e.g., Coenen, Orphanides, and Wieland 2004, Reifschneider and Williams 2000, and Schmitt-Grohé and Uribe 2010). Over the past decade, this pre-crisis consensus has been revised in light of the incoming data (e.g., Blanchard 2014, Chung et al. 2012, Kiley and Roberts 2017, and Williams 2014). Equilibrium real interest rates are now widely expected to recover only to levels that fall short of historical averages—reducing the scope for future cuts in policy interest rates—and disturbances are expected to be larger—increasing the occasional need for such cuts. In the future, policy rates are deemed likely to hit their lower bounds more frequently than previously assumed. Nevertheless, an optimistic view holds that unconventional monetary policies such as quantitative easing and Odyssean forward guidance can be relied upon as substitutes for conventional reductions in policy rates (e.g., Bernanke 2017, Harrison 2017, Kiley 2018, and Reifschneider 2016). Whilst the ZLB may bind from time to time, monetary policy's extended toolkit will rarely be constrained according to this view.[1] But suppose the public are not fully convinced by such assurances. Suppose the economy recovers to a "new normal" (in the terminology of El-Erian 2010) in which people occasionally find reason to worry that policymakers may not be able to respond to future adverse disturbances with sufficient monetary stimulus. How should monetary policy be conducted in such an environment? Are the prescriptions for good monetary policy in "normal times" developed during the Great Moderation sufficient guideposts for determining the appropriate stance of policy?

This paper points to two key differences that set monetary policy in the "new normal" apart from that of the greatly moderated pre-crisis economy. First, when risk is high relative to the available

[1] On a similar note, the view that the Great Recession marked the end of the Great Moderation is disputed; see, for example, Gadea, Gómez-Loscos, and Pérez-Quirós (2018). But even in their analysis, a continuation of the Great Moderation mainly manifests itself in a slow recovery from the Great Recession.

monetary policy space, policymakers should operate the economy above its efficient potential in normal times. This stimulatory bias leans against a tendency for inflation expectations to re-anchor too far below target, but it does allow inflation to settle potentially materially below target in the absence of disturbances. Welfare may be improved by appointing an independent central banker with a slightly higher inflation target then the social optimum. Second, because of constraints on monetary policy alone, changes in the public's perception of risk affect the appropriate stance of monetary policy through time-variation in the appropriate tradeoff between inflation and real stability. A spike in uncertainty, for example, has a negative cost-push effect because of the ZLB alone and makes a given stance of monetary policy less effective. Potentially sizable changes in interest rates may be warranted even if risk does not materialize in any actual disturbance to the economy. This is in sharp contrast to conventional guidelines derived in the optimal monetary literature under conventional perfect foresight assumptions and without considering the ZLB, in which (in the absence of precautionary behavior by households and firms) the appropriate stance of monetary policy is affected only by shocks that have actually occurred or are fully anticipated; see, e.g., Clarida, Galí, and Gertler (1999) and Svensson and Tetlow (2005).

I derive these results in a simple version of the canonical New Keynesian model. The monetary policy design problem for this model served as the "science of monetary policy" for the Great Moderation (Clarida, Galí, and Gertler 1999), and it remains the theoretical foundation for the kind of flexible inflation targeting effectively practiced by major central banks today (Svensson 2010). Throughout, I focus on responses under optimal time-consistent monetary policy. The period-by-period nature of decisionmaking under discretion makes it a realistic description of the actual conduct of monetary policy in a flexible inflation-targeting regime (see, e.g., Bean 2013). In particular, policymakers set interest rates policy meeting by policy meeting to achieve good outcomes given their operational targets. Neither do they follow an instrument rule mechanically, nor do they commit both themselves and future incumbents to a policy plan that will later turn out to be undesirable. In line with conventional wisdom, policymakers cannot bootstrap the economy out of a ZLB episode by promising a future economic boom as advocated by

Eggertsson and Woodford (2003).[2] As noted by Kiley (2018), it is doubtful that any central bank has attempted such purely Odyssean forward guidance in response to binding lower bounds on short-term policy interest rates.

Specifically, I solve for the risky steady state and I study optimal responses to risk shocks around that steady state in a quasi-linear version of the New Keynesian model augmented with a ZLB. In line with the definition in Coeurdacier, Rey, and Winant (2011), the risky steady state is the point at which the economy settles when previous shocks have abated but agents are aware that further shocks may hit in the future. The *risky* steady state differs from the perfect-foresight or *deterministic* steady state, in which agents do not consider the possibility of future shocks. By risk shocks I mean changes in the standard deviations of conventional level shocks in the model. I trace out responses to such changes along the zero-shock path, i.e., the trajectory of the economy over time in which innovations to level shocks do not actually occur. As it were, nothing actually happens in this paper. But crucially, agents remain aware that level shocks could hit the economy at any time in the future and take the economy off the zero-shock path. For analytical tractability, I maintain the assumption in my baseline analysis that agents form expectations at any given point in time in the belief that current risk levels will persist. With this assumption, the effects of risk shocks can be thought of as the economy's responses to changes in a broad notion of the public's perception of risk. A robustness exercise shows how results generalize to the case where agents form fully rational expectations about a stochastic risk shock process as well as about the level shock processes.

I refer to these changes in second moments as risk shocks following the traditional Knightian distinction between risk and uncertainty.[3] Agents in the model economy have well-defined probability

[2]Adam and Billi (2006) show that risk increases welfare gains from time-inconsistent policy plans if policymakers could find ways to credibly commit to them.

[3]While the macroeconomic literature often follow Bloom (2009) in referring to second-moment shocks as uncertainty shocks, this term is also used to describe a range of related phenomena; see, e.g., Kozeniauskas, Orlik, and Veldkamp (2018) for a recent discussion. Fernández-Villaverde et al. (2011) also refer to second-moment shocks as changes in risk, and LeRoy and Singell (1987) discuss the Knightian terminology.

distributions of economic shocks in mind when making decisions, and at any given point in time these distributions coincide with the actual ones. Probabilities are only subjective in the limited sense that agents do not foresee future changes in the levels of standard deviations. We can think of changes in these probabilities as changes in perceived risk only because there is an observational equivalence between the effects of changes in actual and subjective probabilities along the zero-shock path. Elevated risk in the model may well stand in for Knightian uncertainty in reality, but I make no attempt to model such uncertainty about probability distributions directly as, for example, in the work by Ilut and Schneider (2014) and Masolo and Monti (2017).

It is important to note that the macroeconomic risk shocks considered here are very different from the cross-sectional risk shocks analyzed by Christiano, Motto, and Rostagno (2014).[4] In their paper, a "risk shock" refers to a disturbance to the ex post realization of the dispersion of the quality of capital acquired by entrepreneurs. When this dispersion widens, the agency problem associated with financial intermediation becomes more severe. As credit spreads increase, entrepreneurs demand less capital and aggregate demand contracts for a given stance of policy. In the simple New Keynesian model I consider, such a scenario would correspond to a negative *level* shock to the efficient-equilibrium real rate of interest. To keep aggregate demand in line with the economy's supply potential, monetary policy would have to counter cross-sectional risk shocks with a looser policy stance.

The simple quasi-linear structure I consider comes with the benefit that risk affects the economy exclusively through its interaction with the ZLB. Schmitt-Grohé and Uribe (2004) show that risk does not affect decision rules to a first-order approximation, and only the constant term up to a second order, for a general class of models *without* inequality constraints. This class of models includes the textbook New Keynesian model without a ZLB. In my analysis of the New Keynesian model, risk affects decision rules to a first order because of the additional inequality constraint on policy rates—and

[4]Similarly, they are different from the volatility fluctuations considered by Arellano, Bai, and Kehoe (2019).

for that reason only. Second-moment shocks have real effects in the model through expectations if and only if the ZLB is binding in some conceivable states of the world. This allows me to focus on the defining feature of the new environment—an expected regular recurrence of a binding ZLB—and its implications for monetary policy in normal times, without conflating them with effects of risk not stemming directly from a potentially binding ZLB.

I solve the model following the stochastic extended path approach in Evans et al. (2015). On the one hand, the approach follows Adjemian and Juillard (2013) in relaxing the assumption of certainty equivalence maintained in the extended path procedure first proposed by Fair and Taylor (1983). On the other hand, a number of assumptions are imposed to simplify the analysis and focus on the expectational effects of risk directly caused by the ZLB.[5] Compared with a full global solution procedure, the approach relies on (i) a first-order approximation of behavioral relations with the ZLB imposed as the only nonlinearity (through which risks have first-order effects); (ii) the absence of intrinsic persistence (which could occur, for example, because of habits in consumption, indexation in pricing, or capital accumulation); (iii) optimal monetary policy under discretion (so that policymakers do not seek to affect expectations through commitments); and (iv) the approximation of continuous autoregressive shock processes as discrete-space Markov chains. As these assumptions combine to eliminate state variables, the model can be solved in a simple recursive procedure, providing a mapping between risk and macroeconomic outcomes.

The paper is organized as follows. First, section 2 discusses the relation to the literature. Section 3 then describes the model, section 4 the solution method, and section 5 the calibration. Section 6 presents the risky steady state, and section 7 shows responses to risk shocks. Section 8 presents normalization scenarios after a ZLB episode, and section 9 presents an application with stochastic volatility. Finally, section 10 concludes.

[5]These simplifications avoid the curse of dimensionality that forces Adjemian and Juillard (2013) to prune the tree of forward histories when calculating expectations. See Maliar et al. (2015) for a discussion as well as a generalized procedure.

2. Relation to the Literature

The analysis follows previous studies of the implications for discretionary monetary policy of deviations from certainty equivalence in New Keynesian models with a ZLB. Adam and Billi (2007) and Nakov (2008) first showed how the interaction of risk and the ZLB may give rise to a negative bias in expectations in the New Keynesian model's stochastic equilibrium. Both papers illustrate how this bias amplifies the economy's responses to negative shocks to the level of the equilibrium real rate of interest and leads to tradeoffs when monetary policy is driven close to the ZLB. They show how gains from commitment are significantly larger as a result when risk is taken into account. In a recent application, Nakata and Schmidt (2014) further suggest that the skew in expectations provides a justification for a weight-conservative central banker. Similarly, Evans et al. (2015) emphasize that higher risk generally calls for looser monetary policy when the ZLB may bind. They find that liftoff from a ZLB episode should be delayed when agents are concerned about the risk of future episodes.

Compared with these seminal papers, my contribution is, first, to characterize the economy's risky steady state when monetary policy's room for maneuver may be deemed inadequate in normal times. Writing during the "old normal" when ZLB episodes were expected to be rare, Adam and Billi (2007) and Nakov (2008) did not pay close attention to steady-state outcomes or distinguish clearly between different notions of the steady-state and average outcomes. Instead, they emphasized the amplification of effects following a large persistent fall in the equilibrium real rate of interest from its normal level.[6] Second, I trace out dynamics around the risky steady state following changes in risk, illustrating how a risk shock propagates through dynamics in expectations in a way that induces time-varying tradeoffs for monetary policy. I show that this time-variation calls for monetary policy responses even in the absence of actual disturbances (such as level shocks to the equilibrium real rate), and I show that these responses do not depend on the source of risk (i.e., the

[6]Also, Evans et al. (2015) operate in a part of the parameter space for which the solution to the New Keynesian model with a ZLB is explosive, pinned down only by the assumed expectational horizon.

particular shock process for which risk has changed). Moreover, I show that impulse responses to positive and negative risk shocks are asymmetric around the risky steady state because of a nonlinearity in the mapping between risk and economic outcomes.

The simple quasi-linear framework sets my analysis apart from the recent related work by Basu and Bundick (2015, 2017). In these papers, second-moment shocks have real effects through precautionary savings behavior by households. The authors show that these higher-order effects are greatly amplified at the ZLB as monetary policy fails to respond to them.[7] Moreover, a feedback mechanism sets in: monetary policy's inability to respond to what amounts to a further fall in the equilibrium real rate reduces the expected mean of outcomes, in turn inducing further precautionary saving.[8] In my analysis, by contrast, risk shocks affect the economy solely because of changes in expected mean paths for macroeconomic variables for a given equilibrium real rate of interest. The quasi-linearity thus serves both to illustrate that risk shocks may have significant real effects through the ZLB without precautionary behavior and to isolate such direct effects from those effectively operating through shifts in the equilibrium real rate.[9] There are two other significant differences. First, Basu and Bundick (2015, 2017) either impose a simple instrument rule for monetary policy or allow for commitment policies that essentially assume away the effect of the ZLB on mean expectations. Instead, I consider policy prescriptions under the more realistic assumption that optimizing policymakers do not have access to such commitment devices. Second, Basu and Bundick (2015, 2017) are concerned with the contributions of second-moment shocks to macroeconomic volatility over the past. Specifically, they

[7]Basu and Bundick (2015, 2017) compute third-order approximations to behavioral relations to capture effects operating through precautionary behavior when monetary policy is unconstrained, while they turn to global solution methods when imposing the ZLB.

[8]See also Bloom (2009) and Nakata (2017) for discussions of how other nonlinearities may give rise to real effects from movements in risk that may be amplified by the ZLB. Fernández-Villaverde et al. (2015) and Johannsen (2014) suggest that uncertainty about fiscal policy in particular has larger implications when monetary policy is constrained.

[9]See Paoli and Zabczyk (2013) for an analysis of the effect of precautionary saving on the level of the effective equilibrium real rate of interest.

explain how elevated macroeconomic risk could have contributed significantly to the Great Recession, while changes in second moments amounted to inconsequential background noise before the financial crisis. Hence, they distinguish sharply between periods when the ZLB is both nonbinding and a negligible risk, and periods when the ZLB is in fact binding. By contrast, I derive practical normative prescriptions for monetary policy in an environment in which the ZLB is not necessarily binding but nevertheless constantly looming as a cloud on the horizon.

My paper is also closely related to the contemporaneous and independent work by Hills, Nakata, and Schmidt (2016). In their paper, the authors also compare the risky and deterministic steady states in a stylized New Keynesian model, finding results fully consistent with mine. But they then go on to quantify the difference between these steady states in a richer model calibrated to match key features of the U.S. data over the past decades. I instead focus on the risky steady state in a hypothetical new normal, and I proceed to consider the implications of potential time-variation in the underlying level of macroeconomic risk. Moreover, while Hills, Nakata, and Schmidt (2016) consider outcomes under a simple monetary policy rule taken to be representative of monetary policy's reaction pattern in the past, I derive normative prescriptions by solving for optimal monetary policy responses for the future. Finally, while they solve their model in nonlinear form using global methods, I keep my analysis within a quasi-linear framework to facilitate a direct comparison of these prescriptions with those of the influential "science of monetary policy" (Clarida, Galí, and Gertler 1999).[10] I thus take the two papers to be complementary, each providing a different perspective on the interactions between risk and the ZLB.

More broadly, my paper relates to a growing literature on the effects of risk and uncertainty. Following the work of Bloom (2009), there has been a surge of interest in this issue. Whilst the empirical literature has struggled to identify structural risk shocks from the volatility and forecast disagreement measures that are usually taken to be proxies for risk and uncertainty, theoretical work has provided clear channels through which risk shocks may affect the

[10]See, e.g., Carney (2017) for an application to monetary policy in practice.

economy; see, e.g., the survey by Bloom (2014). The expectational mechanism at the core of my analysis is a "bad news channel" in the terminology of Bernanke (1983). It arises because monetary policy is sometimes unable to provide sufficient stimulus in response to large adverse shocks, while it can always act to contract the economy when needed. The model's prediction that the effect of a given risk shock is larger the closer the economy is to the ZLB is in line with the empirical evidence provided by Caggiano, Castelnuevo, and Pellegrino (2017) and Plante, Richter, and Throckmorton (2018). Similarly, the finding that monetary policy is less effective when risk is high is consistent with the evidence in Aastveit, Natvik, and Sola (2017). Finally, Caggiano, Castelnuevo, and Nodari (2018) provide recent evidence that monetary in the United States has indeed responded to changes in aggregate risk.

3. The Model

The model is the canonical New Keynesian model, expressed in log-deviations from its deterministic steady state, extended with a ZLB on interest rates. In addition to a specification of monetary policy, it consists of the following equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

$$x_t = E_t x_{t+1} - \frac{1}{\varsigma}\left(i_t - E_t \pi_{t+1} - r_t^*\right) \tag{2}$$

$$i_t + i^* \geq 0, \tag{3}$$

where E_t is the expectations operator, π_t is inflation at time t in deviation from its target π^*, x_t is the output gap defined as output in deviation from its efficient level, and i_t is the nominal interest rate in deviation from its normal deterministic steady-state value i^*. The first equation is the New Keynesian Phillips curve, the second is the forward-looking IS curve, and the third imposes the ZLB. The model is derived from its microfoundations by Galí (2008) and Woodford (2003) among others.

There are two shock processes in the model. The term u_t is a cost-push process, and r_t^* is the efficient-equilibrium real interest rate in deviation from its steady-state level $r^* \approx i^* - \pi^*$. I assume that the latter is the sum of a deterministic but potentially time-varying component ρ_t and a stochastic process ϵ_t so that $r_t^* = \rho_t + \epsilon_t$. Both the stochastic component of the equilibrium real interest rate and the cost-push shock are given as first-order autoregressive processes with zero-mean Gaussian innovations:

$$\epsilon_t = \mu_\epsilon \epsilon_{t-1} + \nu_{\epsilon,t} \tag{4}$$

$$u_t = \mu_u u_{t-1} + \nu_{u,t}, \tag{5}$$

where $\nu_{\epsilon,t} \sim N(0, \sigma_{\epsilon,t}^2)$ and $\nu_{u,t} \sim N(0, \sigma_{u,t}^2)$. I allow the standard deviations of the innovations to vary over time as indicated by the time subscripts in $\sigma_{\epsilon,t}$ and $\sigma_{u,t}$.

I define a risk shock as a change in one or both of these standard deviations. The standard deviations are independent of each other and can also change independently. As a baseline, however, I consider the special case in which $\varsigma^{-1}\sigma_{\epsilon,t} = \sigma_{u,t} = \sigma_t$ with

$$\sigma_t = \bar{\sigma} + \mu_\sigma \left(\sigma_{t-1} - \bar{\sigma}\right) + \nu_{\sigma,t}, \tag{6}$$

where $\nu_{\sigma,t}$ is an innovation to risk, and $\bar{\sigma}$ is an underlying level of risk in the absence of risk shocks.[11]

Under optimal policy under discretion, a policymaker, hypothetically unconstrained by the ZLB in (3), minimizes the period loss function

$$L \propto \pi_t^2 + \lambda x_t^2 \tag{7}$$

each period subject to the Phillips curve in (1) while taking expectations as given. The loss function can be derived as a quadratic approximation of the utility of the representative household in the full New Keynesian model (again, see, e.g., Galí 2008 and Woodford 2003). The optimality condition takes the form of a conventional targeting rule,

$$\pi_t = -\frac{\lambda}{\kappa} x_t, \tag{8}$$

[11] I only consider realizations of risk that are strictly larger than zero. More broadly, the risk shock process may be specified in logs to rule out nonzero realizations.

stating the optimal policy tradeoff between inflation and the output gap. Following tradeoff-inducing shocks, monetary policy seeks to keep deviations of inflation from target and of output from potential of opposite signs, letting inflation absorb more—and output less—of the adjustment the higher the weight on the output gap in the loss function and the flatter the Phillips curve (i.e., the higher the sacrifice ratio). The interest rate consistent with this optimal allocation can now be found from the IS curve in (2). Since the policymaker is, in fact, constrained by (3), the targeting rule in (8) is replaced by the Kuhn-Tucker conditions

$$(i_t + i^*)\,(\lambda x_t + \kappa \pi_t) = 0 \tag{9}$$

$$i_t + i^* \geq 0 \tag{10}$$

$$\lambda x_t + \kappa \pi_t \leq 0. \tag{11}$$

As shown formally in appendix A, these conditions imply that the interest rate will be set to the maximum of the level consistent with satisfying the targeting rule in (8) and zero.

4. Solution Method

In the main analysis, I assume that agents observe the current level of risk without expecting further changes in this level to occur—even if risk may actually change over time. This assumption simplifies the calculation of the model solution considerably without affecting results qualitatively. In this section, I outline the solution method as well as the definitions of the stochastic steady state and of a simple impulse response function under this simplifying assumption. In section 9, I discuss and present a generalization to the case with stochastic volatility where agents understand the stochastic nature of the risk shock process as well as the processes for the level shocks.

4.1 Solution

I solve the quasi-linear version of the canonical model following the approach in Evans et al. (2015). I approximate the shock processes by independent Markov processes using the Rouwenhorst (1995) algorithm provided by Galindev and Lkhagvasuren (2010). I then solve the model backwards from a distant future period T, beyond which there is no risk and all shocks are zero so that $E_t\pi_{t+1} = E_t x_{t+1} = 0$ for all $t > T$. In each step, I take expectations as given and calculate the unconstrained outcome under each policy regime for a state grid of values for the shock processes, (ϵ, u). I then check if this outcome is consistent with the ZLB in (3) for each node in the grid. If so, I take the unconstrained outcome as the solution for this particular node. If not, I calculate the outcome from the model equations with $i_t = -i^*$ imposed. I then update the ex ante expectations of inflation and the output gap using the Markov transition matrices before progressing to the previous period. The solution consists of the $n_\epsilon \times n_\epsilon$ matrices for inflation, the output gap, and the interest rate, to which this algorithm converges in the initial period $t = 0$. See appendix B for further details.

4.2 Stochastic Steady State

The values for the nodes $(\epsilon = 0, u = 0)$ represent outcomes in the event that no nonzero shock has actually materialized. This converged zero-shock solution at $t = 0$ represents the risky steady state of the model as defined by Coeurdacier, Rey, and Winant (2011). In particular, let $y_0^{sol}(\epsilon, u \mid \sigma)$ be the state-contingent solution for variable $y_t \in \{i_t, \pi_t, x_t\}$ for a given level of risk. The risky steady state for this variable is then defined as $y_0^{sol}(\epsilon = 0, u = 0 \mid \sigma)$. This is the resting point to which the economy returns when all shocks have dissipated. Nonzero realizations of shocks will of course continuously drive the economy away from this point, and the risky steady state potentially differs from the deterministic one exactly because it accounts for agents' expectations that such deviations will occur. Unconditional expectations are averages weighted by unconditional probabilities over outcomes across the state space. Given the Markov structure, these probabilities can easily be derived from the eigenvectors associated with the unitary eigenvalues of the transition matrices; see, e.g., Ljungqvist and Sargent (2000). Appendix C provides further details.

4.3 A Simple Impulse Response Function

I find simple impulse responses to a risk shock by running a double loop. The outer loop moves forward from period $t = 0$, while the inner loop solves the model backwards from period T to the period of the current iteration of the outer loop. For each iteration of the outer loop, I reduce the value of σ_ϵ and/or σ_u from an initial spike according to the assumed process. The economy's responses to the risk shock is the sequence of zero-shock solutions found in the outer loop. In particular, the simple impulse response function is defined as

$$\begin{aligned} I^y_{t+n} &= y_0^{sol}(\epsilon = 0, u = 0 \mid \sigma_{t+n} = \mu_\sigma^n \sigma') \\ &\quad - y_0^{sol}(\epsilon = 0, u = 0 \mid \sigma_{t+n} = \bar{\sigma}) \end{aligned} \tag{12}$$

for $n \in \mathbb{Z}^+$ and where σ' denotes the value of risk on impact of the risk shock in period t.

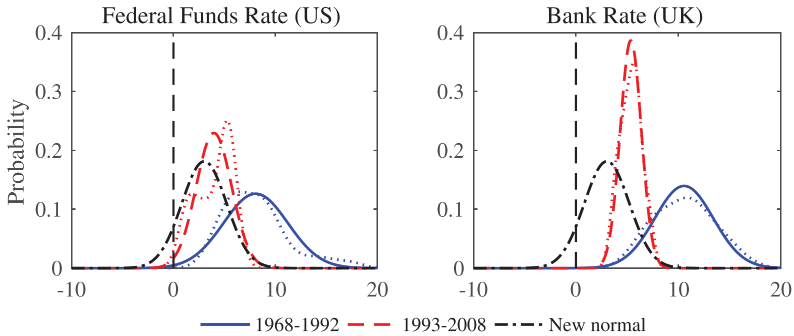
In line with the definition of a traditional impulse response function in Koop, Pesaran, and Potter (1996), this simple function measures the effects of a risk shock hitting the economy at time t on the state of the economy at time $t+n$ given that no other shocks occur. The conceptual experiment is a comparison of the profile for the economy when a risk shock hits with a profile where risk stays at its baseline level, keeping all other shocks (in this case, the level shocks) dormant. But it is a traditional impulse response function with the caveat that agents have myopic rather than rational expectations about the risk shock process itself (while otherwise continuing to form rational expectations). The advantage of taking this approach is that risk does not become a state variable, simplifying calculations considerably. It is also a natural starting point for considering occasional changes in risk perceptions: each period, economic agents simply assign a number to the level of risk they think is present in the economy. But it is a limitation that agents always expect risk to stay constant at a given point in time. As a robustness check, I therefore also show that a generalized specification where agents are allowed to see a stochastic autoregressive profile for risk generates qualitatively similar results (section 9).

5. Calibration

5.1 The New Normal

I calibrate the model to fit a hypothetical "new normal" distribution for desired policy rates—the short-term interest rates a policymaker

Figure 1. Old and New Normal Densities for Policy Rates



Notes: Normal probability density functions with means and standard deviations set equal to sample means and standard deviations for the federal funds rate (left panel) and Bank Rate (right panel) for the sample periods 1968–92 (solid blue lines) and 1993–2008 (dashed red lines) as well as for a hypothetical new normal (dashed-dotted black lines). Dotted lines are estimated kernal distributions with an optimized bandwidth for the normal kernal function for the 1968–92 (in blue) and 1993–2008 (in red) subsamples.

would choose to implement if the ZLB were not a constraint—shown in dashed-dotted black lines in both panels in figure 1. To motivate the choice of this distribution, the figure compares it with recent pre-crisis historical experience in the United States (left panel) and the United Kingdom (right panel). The normal probability density functions shown in solid blue lines share the means and standard deviations with the observed federal funds rate and Bank Rate, respectively, from 1968 through 1992.[12] Blue dotted lines show corresponding kernel density estimates. An observer looking back at these distributions around the time when 2 percent inflation targets were emerging in the early 1990s (see, e.g., Svensson 2010) would not have found much reason to worry about the ZLB. Policy rates had been very volatile over the past quarter of a century (standard deviations were 3.2 percentage points and 2.9 percentage points, respectively), but they had also been high (with means of 8.1 percent and 10.6 percent). The probability that interest rates

[12]For color versions of the figures, see the online version of the paper at http://www.ijcb.org.

should be negative would have seemed negligible. Over the subsequent 15 years, the distributions of policy rates shifted sharply to the left as inflation targeting became established (formally or informally), and inflation expectations anchored at lower levels; see the dashed red lines for normal approximations and dotted red lines for kernel density estimates. In isolation, the lower means (4.0 percent and 5.4 percent) would have increased probability mass below zero. But as volatility fell substantially at the same time, the risk of a binding ZLB did not appear to have increased significantly over the period.

Any Gaussian model informed by interest rate data over the two periods would have produced a probability of negative interest rates very close to zero, including both the semi-structural and the dynamic stochastic general equilibrium (DSGE) models that were increasingly used for such purposes; see, e.g., Chung et al. (2012), Coenen, Orphanides, and Wieland (2004), and Schmitt-Grohé and Uribe (2010). Moreover, assessments of the fit of models estimated using full-information techniques—for example, by comparing fitted normals with kernel distributions (a tougher test than usually applied)—would have pointed to a satisfactory if not perfect fit.[13] It is therefore not surprising that a consensus emerged in the pre-crisis period that the ZLB should be of no great practical concern. As it turned out, however, the estimated models were too sanguine about prospects of a binding ZLB.

Instead of focusing on past distributions, I therefore consider a hypothetical scenario for the normal for monetary policy to which the economy may recover after the financial crisis. Compared with the historical distributions over the 1993–2008 period, it is defined by a further shift to the left and an increase in the spread for desired policy rates. The shift captures an assumed decline in the trend component of the efficient equilibrium real rate of interest. The higher spread reflects an end to the Great Moderation so that

[13]The sample periods in figure 1 are such that estimated distributions are close to normal without detrending. In particular, Bank Rate is very close to normal in each of the two subsamples. The picture is a bit less clear for the United States, where the data point as much to a gradual decline as to a single structural shift in the level around which the federal funds rate fluctuates. The sharp response to the 2001 recession followed by rapid normalization also results in a near-bimodal shape of the estimated kernel for the federal funds rate over the 1993–2008 period.

larger disturbances call for stronger policy responses then in the pre-crisis inflation-targeting period. I set the average desired policy rate to 3 percent—in line with the forecasts emphasized by Reifschneider (2016) and the recent estimates in Del Negro et al. (2017), Kiley (2015), and Laubach and Williams (2016)—and I fix the standard deviation at 2.2, in between the values for the two pre-crisis subsamples for the United States and the United Kingdom. Hence, macroeconomic risk is higher than during the Great Moderation, but not as high as in the preceding decades. With the lower average level of the policy rate, the moderate increase in the desired spread increases probability mass below zero to about 9 percent. In this "new normal" scenario, therefore, the probability that policymakers would set policy rates to negative values if they could is clearly non-negligible.[14]

5.2 Parameterization

In the baseline calibration, I parameterize the model so that it features a distribution for the unconstrained optimal policy rate corresponding to the new normal distribution in figure 1. The (annualized) inflation target is assumed to be $\pi^* = 2\%$ and the deterministic steady-state level of the real interest rate $r^* = 1\%$. The normal nominal interest rate is then approximately $i^* = 3\%$ with a discount factor $\beta = 0.9975$. The deterministic component of r_t^* is set to $\rho_t = 0$ for all t, and the inverse of the elasticity of intertemporal substitution to $\varsigma = 1$ as is common in the literature for this poorly identified parameter (see, e.g., Galí 2008). The slope of the Phillips curve is assumed to be $\kappa = 0.02$. This value is at the lower end of the 0.02–0.05 range of empirical estimates collected by Woodford (2005), in line with the hypothesis that Phillips curves have (if anything) flattened in recent decades (e.g., Blanchard, Cerutti, and Summers 2015).

The weight on the output gap in the loss function is similarly set to $\lambda = 0.02$. This is larger than the $\lambda = \kappa/\zeta$ imposed when the loss function is derived from household utility in the basic

[14]The assumption that desired policy rates are normally distributed is not restrictive. What matters for the qualitative results in the following is that probability mass below zero is non-negligible more than the specific shape of the distribution.

New Keynesian model, where $\zeta > 1$ is the elasticity of substitution between product varieties under monopolistic competition. But for conventional values of ζ around 6 and empirically plausible values of κ, the weight on output would be much smaller than actual mandates for monetary policy seem to imply. The assumed value of λ corresponds to a weight on output stabilization of about a third in annualized terms, a reasonable interpretation of the degree of flexibility in inflation targeting in practice (see, e.g., Carney 2017, English, López-Salido, and Tetlow 2015, and Svensson 2010). Moreover, the basic New Keynesian model is likely to underestimate the appropriate weight on the output gap; see, e.g., Debortoli et al. (2016) and Walsh (2014). With an implied targeting rule for monetary policy with a slope of -1, policymakers seek to let quarterly inflation and the output gap share the burden of adjustments to tradeoff-inducing disturbances equally.

Level shocks are assumed to be moderately persistent with $\mu_u = 0.25$ and $\mu_\epsilon = 0.75$. With these parameter values, an underlying level of risk given by $\sigma = 0.2725/100$ delivers a standard deviation of the unconstrained nominal policy rate of 2.2 when $\varsigma^{-1}\sigma_{\epsilon,t} = \sigma_{u,t} = \sigma_t$ so that the desired policy rate is negative with probability 9 percent. For comparison, I also consider a low-risk scenario with $\sigma = 0.1234/100$. With this low level of underlying risk, the dispersion of desired interest rates is kept low while the mean shifts down to 3 percent. Specifically, the standard deviation of desired interest rate is 1 percentage point, similar to the level observed for the United Kingdom between 1993 and 2008. The probability that interest rate should be negative remains negligible in this case despite a low level of r^*. In section 7, I show impulse responses to risk shocks in this low-risk scenario as well as in the baseline new normal.

Finally, I solve the model with an expectational horizon of $T = 1{,}000$. The solution algorithm converges with significantly fewer iterations than 1,000. Hence, results are not sensitive to this choice of T.

5.3 Model Fit

Table 1 compares the "new normal" baseline and the low-risk scenarios with alternative calibrations in which σ is set to fit the

Table 1. Alternative Calibrations

Episode	Data				Unconstrained Model					P_0
	$E(i)$	$\sigma(i)$	$E(\pi)$	$\sigma(\pi)$	$E(i)$	$\sigma(i)$	$E(\pi)$	$\sigma(\pi)$	100σ	
New Normal	—	—	—	—	3.02	2.20	2.00	2.48	0.27	0.09
Low Risk	—	—	—	—	3.02	1.00	2.00	2.11	0.12	0.00
US 1968–1992	8.07	3.16	5.96	3.73	8.07	3.16	4.16	4.66	0.39	0.01
US 1993–2008	3.97	1.74	2.55	3.59	3.97	1.74	2.00	2.31	0.22	0.01
UK 1968–1992	10.56	2.86	8.77	6.83	10.56	2.86	6.56	6.83	0.35	0.00
UK 1993–2008	5.36	1.03	1.93	2.09	5.36	1.03	2.00	2.11	0.13	0.00

Notes: $E(.)$ and $\sigma(.)$ denote means and standard deviations, respectively, of the nominal interest rate and inflation. Interest rates are measured by the FFR and BR for the United States and United Kingdom, respectively. Inflation is annualized quarterly CPI inflation (source: Datastream). P_0 denotes the probability of negative interest rates in the unconstrained model as well as in normal distributions with means and standard deviations as in the data.

distribution of the unconstrained policy rate to the historical distributions in figure 1. In the historical episodes, I match the mean by assuming that $r^* = 3.75$ in the 1968–92 period, and that $\pi^* = 2\%$ between 1993 and 2016. In all of the alternative calibrations, the probability of negative interest rates is close to zero, rounding to 1 percent only for the United States. Even if the model is too stylized to capture the covariance structure of the data more broadly, the table shows that it gives a fairly good fit to observed inflation volatility across the historical periods. Hence, it does not seem unreasonable a priori that it can shine some light on inflation outcomes in hypothetical scenarios. Nevertheless, I make no claim to provide empirical accuracy in this simple framework. The numerical results should be taken as qualitative indication of the sign and order of outcomes rather than quantitative estimates.

In what follows, I assume that the lower bound is exactly zero as specified in (3). The unconstrained model is hardly a good guide for monetary policy when the desired level of interest rates is negative with a non-negligible probability as in the hypothetical new normal. Absent substantial reform to the payment system, it is improbable that policymakers can persistently drive interest rates into negative territory (e.g., Rogoff 2015). It is more likely that unconventional policies such as quantitative easing may act as substitutes for negative short-term interest rates (e.g., Haldane et al. 2016). The interest rate in the model may best be thought of as a *shadow* rate implicitly incorporating the effects of unconventional policy tools (Black 1995). The shadow rate may be negative if these tools are operational whenever the ZLB binds. But if either the availability or the effectiveness of such tools is limited, the shadow rate will also be bounded from below at some level less than zero. With the ZLB imposed, I effectively assume that cash has not been phased out to allow for negative interest rates, and that unconventional policies cannot act as perfect substitutes for negative interest rates. The analysis becomes irrelevant if either of these assumptions is fully reversed so that interest rates can go negative without difficulty or side effects, or unconventional policy tools can be relied upon as perfect substitutes for changes in policy rates. But the conclusions hold for any combination of the effective lower bound and the distribution of the desired shadow rate such that monetary policy is constrained with a non-negligible probability.

I take such a constellation to be the defining feature of the new normal.

6. Risky Steady State

6.1 Baseline

The first row in table 2 shows risky steady-state outcomes for the interest rate (i), inflation (π), and the output gap (x) in the full model with the ZLB under the baseline calibration. In the new normal, public perceptions of risk are high enough that, because of the ZLB, policymakers are not expected to be able to respond sufficiently to some of the large negative disturbances that are deemed likely to hit the economy in the foreseeable future. By contrast, the public know that monetary policy can always be tightened appropriately, also in response to large inflationary shocks. This asymmetry introduces a negative skew in expectations as first emphasized by Adam and Billi (2007) and Nakov (2008). Under optimal discretionary policy, policymakers lean against the tendency for inflation expectations to anchor below target by operating the economy above potential in normal times through a stimulatory bias in policy rates. Such a long-run effect on real output is feasible because the monetary policy stance interacts with inflation expectations to determine real interest rates in the risky steady state. In this sense, monetary policy is no longer neutral in the new normal's long run. But there are limits to policymakers' willingness to overheat the real economy, and subdued inflation expectations are allowed to weigh on the price setting of firms. Consequently, inflation settles about 20 basis points below target.

Hence, the point at which the economy comes to rest when shocks have faded away does not coincide with the deterministic steady state, in which inflation is on target, the output gap is closed, and the interest rate is at its normal level. Moreover, as table 2 also shows, unconditional expectations deviate from steady-state values. In expectation, output falls short of potential because of spells at the ZLB. These episodes drive the average interest rate above its risky steady-state level, and expected inflation falls somewhat further below target. Notice also that, because of monetary policy's inability to deliver the desired stimulus, the frequency of ZLB

Table 2. New Keynesian Model with ZLB under Optimal Discretion

Scenario	Interest Rate			Inflation			Output Gap			P_{ZLB}
	i^*	i	$E(i)$	π^*	π	$E(\pi)$	x^*	x	$E(x)$	
New Normal	3.02	2.73	2.81	2.00	1.80	1.79	0.00	0.05	−0.01	0.14
r^* Risk Only	3.02	2.94	2.94	2.00	1.93	1.92	0.00	0.02	−0.00	0.07
u Risk Only	3.02	2.98	2.99	2.00	1.98	1.97	0.00	0.01	−0.00	0.04
Lower r^*	2.76	2.25	2.40	2.00	1.66	1.64	0.00	0.09	−0.01	0.20
Lower π^*	2.77	2.27	2.43	1.75	1.43	1.41	0.00	0.08	−0.01	0.20
Higher r^*	3.27	3.10	3.14	2.00	1.88	1.87	0.00	0.03	−0.01	0.10
Higher π^*	3.27	3.09	3.14	2.25	2.12	2.11	0.00	0.03	−0.00	0.10
Very High π^*	5.04	5.03	5.03	4.00	3.99	3.99	0.00	0.00	−0.00	0.01

Notes: i, π, and x are risky steady-state values, stars denote deterministic steady-state values, $E(.)$ and $\sigma(.)$ denote means and standard deviations, respectively, and P_{ZLB} denotes the frequency of a binding ZLB. Deterministic steady-state values for interest rates and inflation satisfy $1 + i^*\% = (1 + r^*\%)(1 + \pi^*\%)$.

episodes is higher than the probability that interest rates should be negative in the unconstrained model. In the new normal, a 9 percent probability that desired policy rates are negative translates into a 14 percent probability that the ZLB is binding.

6.2 Sensitivity to Risk and Policy Space

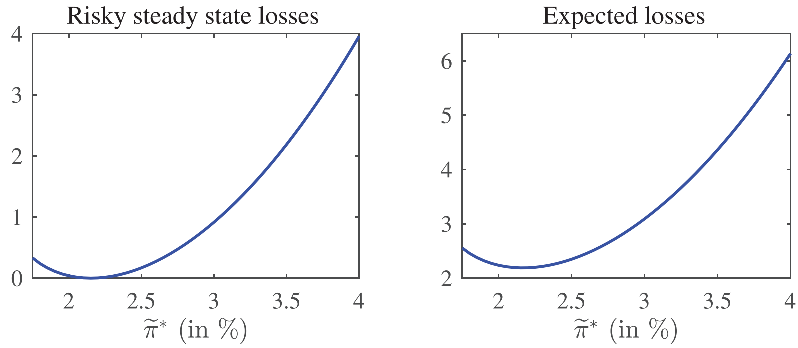
The remaining rows in table 2 illustrate the sensitivity of these statistics to assumptions about risk and the available monetary policy space. In the baseline, I assume that $\varsigma^{-1}\sigma_{\epsilon,t} = \sigma_{u,t} = \sigma_t$ for convenience. But in general, the levels of risk for the two shock processes may not be related by a simple multiple. The second row shows the effect of completely removing the risk of cost-push shocks, while keeping the risk of r^* shocks at the baseline value. The third row shows the opposite case without a perceived risk of r^* shocks. In both cases, the risky steady state deviates from the deterministic one with inflation settling below target. The marginal contributions of the two shocks are similar, but the deviations are much smaller with inflation rates of 1.93 and 1.98, respectively. In the new normal, agents are particularly concerned about the inability of policymakers to respond when large adverse disturbances to the cost-push process and the equilibrium real rate coincide.

Higher risk for individual shocks may, however, result in similar biases as in the benchmark.[15] Inflation may fall short of target in the new normal regardless of the source of risk. Notice also that policymakers have substantial room for maneuver. Inflation falls short of target in normal times only because private agents worry that the ZLB may bind in the future. The re-anchoring of inflation expectations occurs whenever risk is perceived to be high relative to the available monetary policy space.

For a given level of risk, the effect on expectations therefore also depends on the normal distance to the ZLB, as illustrated in the remaining cases in table 2. The closer the economy operates to the ZLB, the larger are the effects of risk on outcomes. If the distance to

[15]Increasing σ_ϵ to about 0.0032 when the risk of cost-push shocks is absent, or σ_u to about 0.0043 when the risk of r^* shocks is negligible, leads to similar biases in inflation and the output gap as under the baseline calibration.

Figure 2. Welfare Losses as a Function of Inflation Target



Notes: Period welfare losses with an optimal annual inflation rate of 2 percent as a function of an operational inflation target denoted by $\tilde{\pi}^*$. The left panel shows losses evaluated in the risky steady state, and the right panel shows unconditionally expected losses.

the ZLB is reduced by about 25 basis points, either because the equilibrium real rate of interest is lower (row 4), or because monetary policy targets a lower inflation rate (row 5), risky steady-state inflation falls below target by a further 15 basis points. When inflation in the deterministic steady state itself is lower, inflation settles almost 40 basis points lower than in the baseline. By contrast, if i^* increases to about 3.25 percent, the negative bias in inflation is reduced by 7–8 basis points. With a higher inflation target, the component of i^* that can be chosen by policymakers, inflation settles around 2.1 percent (row 7). To fully eliminate the negative bias in inflation, however, policymakers will have to target a rate of inflation above 4 percent (row 8).

6.3 Welfare and the Inflation Target

Since inflation is closer to 2 percent and the output gap is closer to zero when monetary policy targets an inflation rate of 2.25 percent (as in row 7 in table 2), welfare losses are unambiguously lower in the risky steady state when evaluated using a loss function (7) that is centered around a 2 percent optimal rate of inflation. In the new normal, welfare may be improved by appointing an independent central banker with a slightly higher operational inflation target than the social optimum. But as shown in figure 2, setting the operational target too high—for example, at the level which eliminates the negative bias in expectations—comes with substantial welfare costs if the socially optimal rate of inflation is 2 percent. Specifically, only operational targets in the open interval (2.00%, 2.30%) result in smaller welfare losses both in the risky steady state and in expectation, where expected period welfare losses are calculated as the probability-weighted sum of losses across the state space. Provided that monetary policy responds optimally, the simple analysis does not by itself provide a case for increasing the inflation target to, say, 3 percent or 4 percent as suggested as a potential response to low equilibrium interest rates, e.g., by Ball et al. (2016), Blanchard, Dell'arricia, and Mauro (2010), Krugman (2014), and Williams (2009).

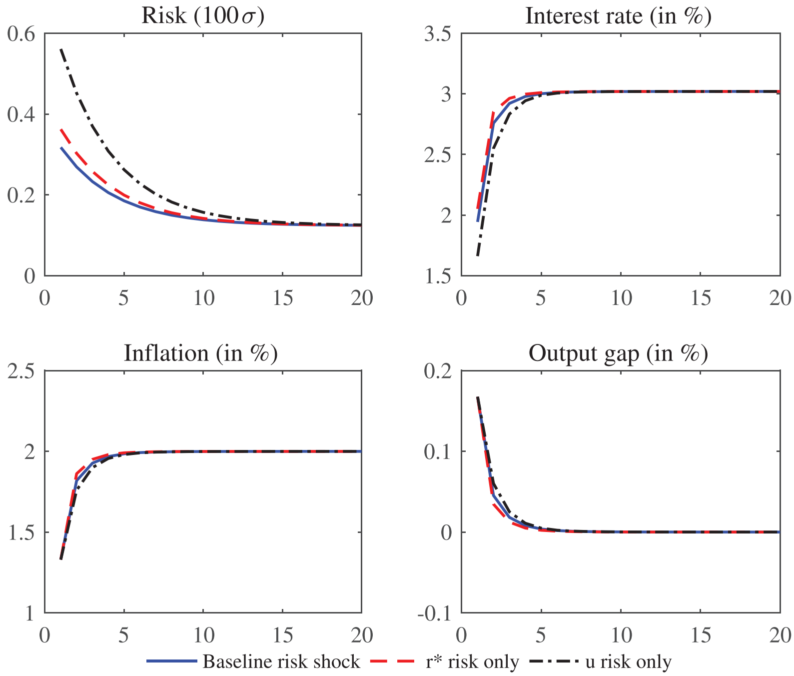
7. Impulse Response to Risk Shocks

Now suppose that risk may vary over time. How does the economy adjust to changes in the perception of risk? To build intuition, I first present simple impulse responses to the baseline risk shock in (6) starting from a low-risk steady state. The low-risk case differs from the new normal only in that $\sigma = 0.0012$. With this low level of underlying risk, the probability that interest rates should be negative remains negligible despite a low level of r^* (see the second row in table 1). The risky steady state therefore practically coincides with the deterministic steady state. I then consider a risk shock for each of the two level shocks in turn, before turning to a baseline risk shock in the risky steady state implied by the new normal scenario.

7.1 The Case of Low Underlying Risk

Solid blue lines in figure 3 are impulse responses to the baseline risk shock along the zero-shock path starting from the low-risk steady state. The risk shock represents a scenario in which risk is temporarily elevated so that agents expect shocks to be drawn from distributions with higher spreads for some time in the future. But the economy is not actually hit by any level shocks along this adjustment path; it is only the perception of risk that changes. When risk spikes up, agents begin to worry about the monetary policymaker's inability to respond to large adverse shocks as a consequence of

Figure 3. Impulse Responses to Risk Shocks



Notes: Impulse responses to a baseline risk shock (solid blue lines), a shock to r^* risk only (dashed red lines), and to cost-push risk only (dashed-dotted black lines), around a low-risk steady state ($\sigma = 0.0012$) in the canonical New Keynesian model with a ZLB on interest rates under optimal discretionary monetary policy.

the ZLB. Therefore, inflation expectations fall short of the inflation target, and output expectations of potential. By (1), the risk shock has a negative cost-push effect: for any given level of the output gap, inflation falls in response to lower inflation expectations. This effect induces a tradeoff for the policymaker as reflected in the targeting rule in (8). Under optimal discretion, the policymaker loosens policy enough to bring output above its efficient potential. The expansion in the economy works to limit the fall in inflation and appropriately balance deviations from target with real economic outcomes. As risk falls back, the ZLB becomes less of a concern and the economy gradually returns to the low-risk steady state.

The dynamics induced by the risk shock are similar to those following a level cost-push shock in the New Keynesian model. But with a risk shock, the interest rate has to be reduced more to achieve the optimal balance between inflation and the output gap. There are two reasons for this. First, lower inflation expectations raise the real interest rate for a given level of the nominal rate. And second, since output expectations have also been adversely affected by the risk shock, policy needs to bring about a lower real interest rate to boost aggregate demand through (2). In this sense, the increase in risk has made monetary policy less effective.

Importantly, a tradeoff arises in uncertain times even if shocks do not actually happen. The only prerequisite is that the risk shock is large enough that the ZLB becomes a concern. Small increases and reductions in risk around the low-risk steady state leave economic outcomes unaffected. Of course, the closer the economy operates to the ZLB, the more risk shocks become "large" in this sense. Reversely, if underlying risk is high, the tradeoff for monetary policy becomes a permanent feature of the economy as in the new normal described above.

7.2 On the Sources of Risk

Figure 3 also shows the effects of a positive risk shock around a low-risk steady state for each of the two shocks in turn. Again, the standard deviations for the two shock processes move together in the baseline risk shock mainly for convenience. There is no reason to rule out a priori that risk cannot move independently for the two types of shocks. Qualitatively, however, the economy responds in the same way to the two individual shocks. Spikes in risk lead to cost-push effects both when risk is elevated for r^* only (dashed red lines) and for the cost-push process only (dashed-dotted black lines). It is simply the numerical increases in risk required to induce similar quantitative dynamics that are different (top-left panel). In both cases, responses are driven by an increase in the likelihood that policymakers cannot provide sufficient stimulus. But the sources of the potential adverse shocks are immaterial. For cost-push shocks, a negative bias in inflation expectations occurs because monetary policy cannot always engineer a sufficient boom in the economy to prevent inflation from falling too much after large negative shocks. In the case of r^* shocks, sufficiently negative realizations make it impossible for monetary policy to provide enough support for aggregate demand to keep up with supply.[16] A tradeoff arises for monetary policy as the prospect of such demand-driven recessions feed into inflation expectations when risk is elevated.[17]

Notice, however, that shocks to r_t^* are not necessarily demand shocks in the traditional sense. In the canonical New Keynesian model, fluctuations in the efficient equilibrium real rate of interest are driven by changes in the expected growth rate of total factor productivity in addition to changes in preferences and exogenous spending; see, e.g., Galí (2008). Heightened uncertainty about the future growth potential of the economy is therefore an example of a risk shock to r_t^*. A scenario in which such an increase in perceived risk is associated with a fall in expected future growth rates would correspond to a combination of a positive risk shock and a negative level shock to r^* in this framework.

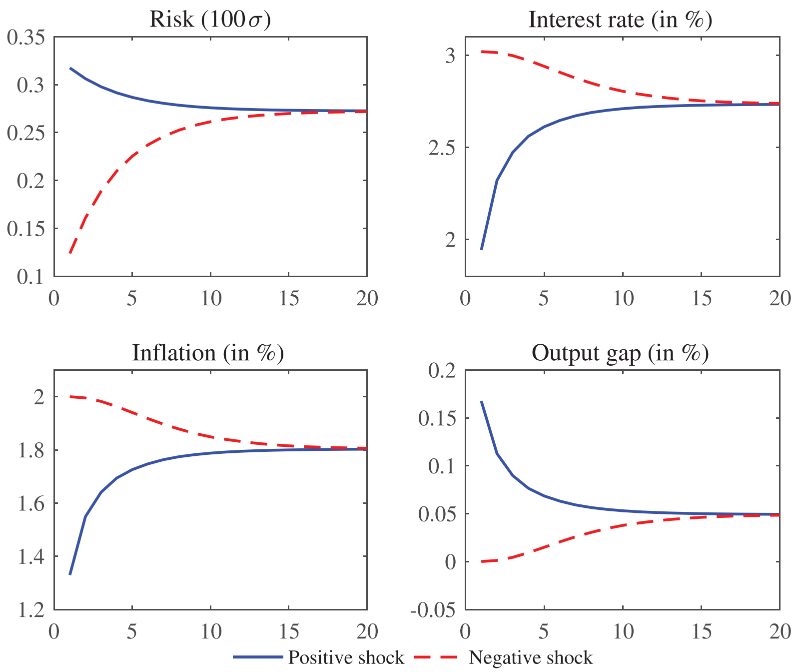
7.3 Risk Shocks in the New Normal

Starting from the new normal steady state, both positive and negative risk shocks have cost-push effects as shown in figure 4. Responses to a positive shock (solid blue lines) are as before, except that the economy reverts to the risky steady state with a negative bias in inflation. An increase in risk increases the bias in expectations, worsening the tradeoff for monetary policy. But a negative risk shock (dashed red lines) now has a positive cost-push effect. As risk falls, agents stop worrying about the ZLB, and inflation expectations realign with the inflation target. Policymakers increase interest rates in response, while the output gap closes. Gradually, as risk returns to its underlying level, the economy reverts to the high-risk steady state. As the responses show, optimal monetary policy in the

[16] If monetary policy were unrestricted by the ZLB, shocks to r_t^* could always be perfectly offset by an appropriate stance of policy. In this case, the output gap would remain closed, and inflation would be on target by the divine coincidence (Blanchard and Galí 2007).

[17] As illustrated by Adam and Billi (2007), a tradeoff arises for persistent negative *level* shocks to r^* of an intermediate size for a similar reason: when the economy moves closer to the ZLB, more future shocks can potentially cause a recession for a given level of risk.

Figure 4. Impulse Responses to Risk Shocks in New Normal

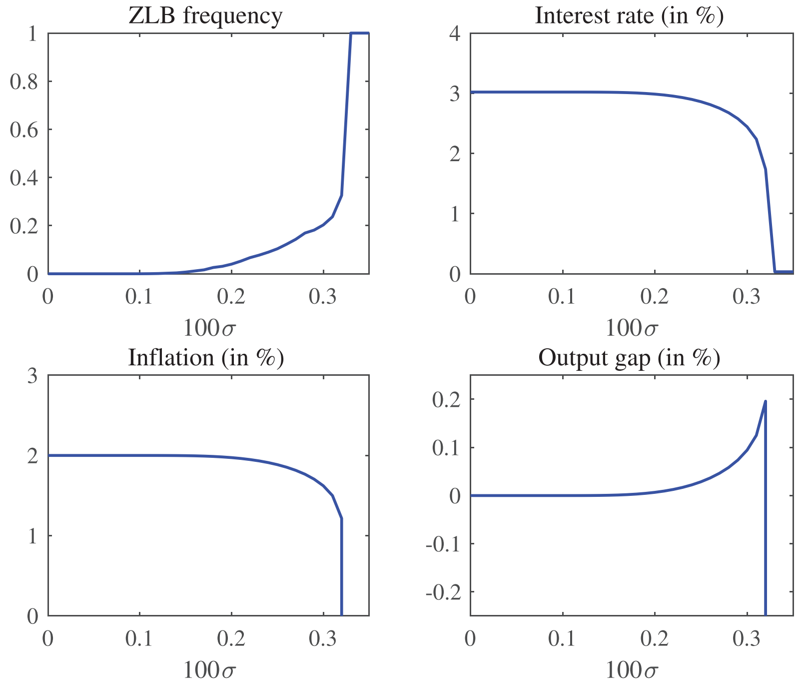


Notes: Impulse responses to a positive (solid blue lines) and a negative (dashed red lines) baseline risk shock around the new normal risky steady state (σ = 0.0027) in the canonical New Keynesian model with a ZLB on interest rates under optimal discretionary monetary policy.

new normal responds nimbly when risk perceptions change in both directions.

The asymmetry in the responses to positive and negative risk shocks around the risky steady state reflect a nonlinearity in the effect of risk on economic outcomes as illustrated in figure 5. With low levels of risk, the economy operates in the deterministic steady state in the absence of level shocks. As risk increases, the ZLB eventually becomes binding in some states of the word. For small increases, the effects are small. But as risk increases further, the frequency of ZLB episodes increases and effects begin to accelerate. Beyond a certain critical point (around σ = 0.0032 under

Figure 5. Economic Outcomes as a Function of Risk

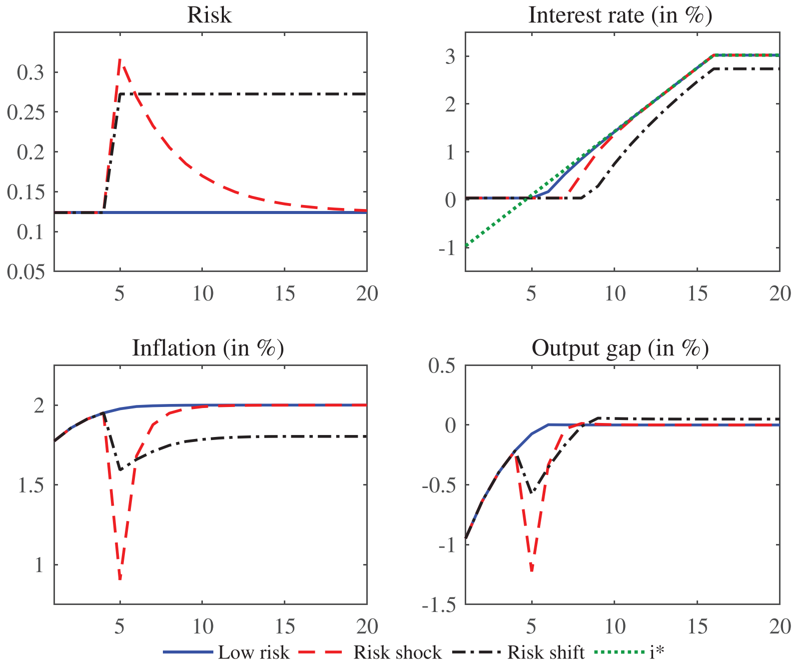


Note: Economic outcomes as a function of risk in the canonical New Keynesian model with a ZLB on interest rate under optimal discretionary policy.

the baseline calibration), interest rates are driven to the ZLB by the bias in expectations itself. In this unpleasant scenario, negative expectations—caused by a concern about the policymaker's inability to respond to adverse shocks—become self-fulfilling as the policymaker is, in fact, unable to respond sufficiently to these expectations because of the ZLB. As a result, the economy enters a downward spiral with hyperdeflation and a collapse of output.[18]

[18]The possibility of explosive dynamics corresponds to the potential nonexistence of equilibriums analyzzed by Mendes (2011) and Nakata and Schmidt (2014).

Figure 6. Normalization Scenarios



Note: Recovery from a ZLB episode as reflected in the path of the equilibrium interest rate (dotted green lines in the top-right panel) under optimal discretionary policy in a low-risk scenario (solid blue lines), in a low-risk scenario with a baseline risk shock (dashed red lines), and with a shift in risk (dashed-dotted black lines).

8. Normalization Scenarios

To illustrate the implications of a binding ZLB for the propagation of risk shocks, figure 6 shows a normalization scenario in which the economy is gradually recovering from a ZLB episode caused some time in the past by a large and persistent negative shock to the level of the equilibrium real interest rate. The nature of this initial shock—say, a financial crisis—is well understood by agents in the economy by now. Specifically, the deterministic component driven by ρ_t is known to follow the path shown in the top-right panel of figure 6 (dotted green line) so that the equilibrium nominal interest rate gradually returns to a new normal level of 3 percent. Uncertainty surrounding this recovery is perceived to be low ($\sigma = 0.0012$).

At around period $t = 4$, the efficient nominal interest rate turns positive and the policymaker, who operates under optimal discretion, is preparing to lift interest rates off the ZLB. In the absence of risk, the policymaker would simply follow the equilibrium interest rate on its trajectory back toward normal levels once it exceeds the ZLB.[19] But as long as the equilibrium interest rate is this close to the ZLB, even small shocks are "large," and the possibility that a shock may drive the economy back to the ZLB in the future is sufficient to optimally delay liftoff even when risk is low.[20]

Now suppose that agents suddenly become more uncertain about economic prospects, perhaps reflected in turmoil across financial markets. Specifically, suppose the economy is hit by a baseline risk shock corresponding to the one shown in figure 3 at time $t = 5$, just as liftoff was supposed to take place in the absence of any disturbances to the economy. Now that the economy is close to the ZLB, the impact effect of the risk shock on expectations is larger than before, as the monetary policymaker is constrained by the ZLB in its response to the shock. As shown in figure 6 (dashed red lines), inflation falls more as a consequence, and liftoff from the ZLB is further delayed. Now because of the binding ZLB, output also falls further below potential. Only as risk abates will the optimal interest rate path catch up with the equilibrium rate. The longer risk stays elevated, i.e., the more persistent the risk shock, the longer liftoff is optimally delayed even if the economy is not actually exposed to any shocks during the recovery.

Following this temporary temporary risk shock, the economy eventually returns to a low-risk steady state with inflation on target. If the shock instead takes the form of a permanent increase in underlying risk to the level associated with the new normal, the economy instead gradually settles in the risky steady state as shown in dashed dotted black lines. In this normalization scenario, optimal

[19]This corresponds to the perfect foresight case analyzed by Adam and Billi (2007) and Guerrieri and Iacoviello (2015).

[20]This is the argument made in Evans et al. (2015). But in figure 6 the ZLB binds because of an initial level shock to the equilibrium real rate of interest and not, as in their analysis, because of an explosively high risk level that may keep the economy at the ZLB for an arbitrary length of time depending on the expectational horizon.

policy lifts off from the ZLB late and continues to lean against low inflation expectations. The optimal tradeoff, however, requires the policymaker to accept that that inflation settles below below target as the economy recovers to its new normal.

9. Stochastic Volatility

So far, I have maintained the assumption that agents form expectations at any given point in time in the belief that current risk levels will persist. This assumption has allowed me to illustrate how risk interacts with the ZLB in the simplest possible framework. I now show how the results generalize to a setting in which agents understand the stochastic nature of the risk shock process.

The generalization comes at the cost of some computational complexity. As for the level shocks, I approximate the risk shock process in (6) by an independent Markov process. I assume that risk shocks are drawn first in each period, followed by the level shocks given the realization of risk. This assumption allows me to calculate one-period-ahead expectations across a three-dimensional state grid and solve the model using a generalization of the iterative procedure outlined in section 4. Specifically, I fix the state space for level shocks and calculate transition probabilities for a each level of risk using the approach in Tauchen (1986).[21] In each iteration of the solution procedure, I can find state-contingent one-period-ahead expectations conditional on the level of risk as before. In an additional step, I can now find the unconditional one-period-ahead expectations for each node in the grid as a sum of conditional expectations across risk levels weighted by transition probabilities for risk. Appendix E presents this extension to the solution method in Evans et al. (2015) in more detail.

Table 3 shows steady-state outcomes when the standard deviation of the innovation to the risk shocks process, $\nu_{\sigma,t}$, is set to

[21] I am grateful to an anonymous referee for suggesting this approach. In what follows, the Tauchen multiple parameter is set to $m = 3$ and the size of the grid for the risk shock process is $n_\sigma = 9$.

Table 3. New Keynesian Model with ZLB and Stochastic Volatility

Scenario	Interest Rate			Inflation			Output Gap			
	i^*	i	$E(i)$	π^*	π	$E(\pi)$	x^*	x	$E(x)$	P_{ZLB}
$100\sigma = 0.2668$	3.02	2.70	2.79	2.00	1.78	1.77	0.00	0.06	−0.01	0.14
$100\sigma = 0.2725$	3.02	2.65	2.76	2.00	1.75	1.74	0.00	0.06	−0.01	0.15

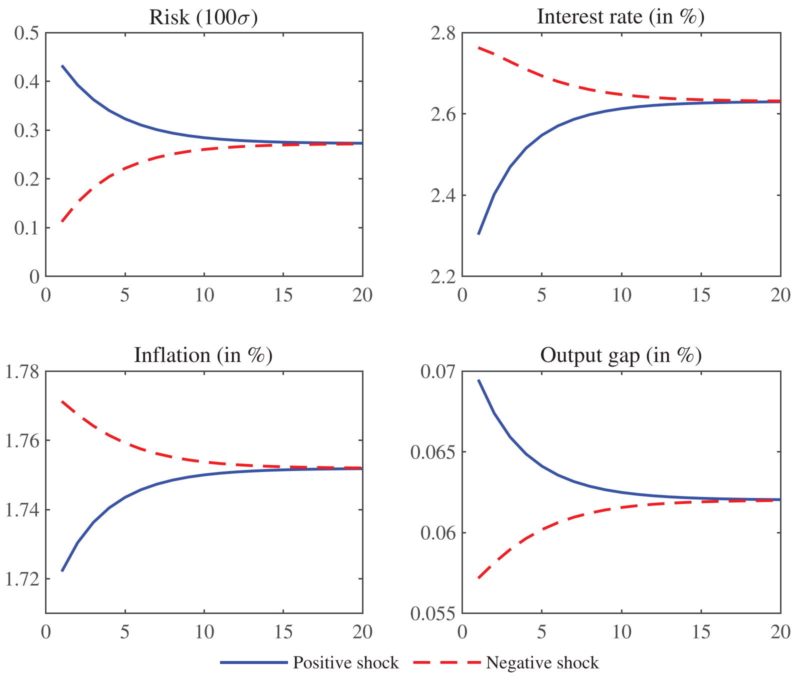
Notes: i, π, and x are risky steady-state values, stars denote deterministic steady-state values, $E(.)$ and $\sigma(.)$ denote means and standard deviations, respectively, and P_{ZLB} denotes the frequency of a binding ZLB. Deterministic steady-state values for interest rates and inflation satisfy $1 + i^*\% = (1 + r^*\%)(1 + \pi^*\%)$.

0.05/100.[22] In the first row, the underlying level of risk, $\bar{\sigma} = 0.2668/100$, is chosen so that the unconstrained model with stochastic volatility also matches the volatility of the policy rate in the hypothetical new normal discussed in section 5. Results are similar to those for the new normal considered in section 6. But despite a lower level of $\bar{\sigma}$, the bias in expectations is slightly larger and inflation settles about 2 basis points further below target in the stochastic steady state when the ZLB is imposed. In the second row, $\bar{\sigma} = 0.2725/100$ as in the new normal calibration of the model without stochastic volatility. As agents now take account of the stochastic nature of risk, inflation settles at $\pi = 1.75\%$ in the stochastic steady state, a further 5 basis points below target compared with the case with constant risk. For a given level of underlying risk, agents now find it more likely that monetary policy may be constrained in the future, as occasional spikes in risk are expected to result in larger adverse level shocks.

Figure 7 shows generalized impulse responses to a positive and a negative baseline risk shock with stochastic volatility when $\bar{\sigma} = 0.2725/100$. By contrast to the simple impulse responses shown in section 7, these impulse responses are derived under the assumption that agents expect the level of risk to follow the profile shown in the upper-left panel. Agents now have fully rational expectations about all shocks in the model. See appendix E for a precise definition. Qualitatively, the generalized impulse responses are as the same as the simple ones presented in section 7. A positive risk shock has a negative cost-push effect, and the policymaker loosens policy to limit a fall in inflation. A negative risk shock has a positive cost-push effect, and the policymaker tightens policy to limit an increase in inflation. As before, the effects of the risk shocks are asymmetric. But now that agents expect any risk shock to be temporary, the effects of a shock of a given size are much smaller. When agents expect risk to return to its underlying level according to the risk shock process, monetary policy is only expected to be constrained more often within a relatively short horizon. The effect on expectations is therefore significantly smaller. With fully rational expectations, therefore,

[22]This standard deviation ensures that risk is positive in all states for the levels of underlying risk considered.

Figure 7. Generalized Impulse Responses to Risk Shocks



Note: Impulse responses to a positive (solid blue lines) and a negative (dashed red lines) baseline risk shock around a risky steady state ($\sigma = 0.0027$) in the canonical New Keynesian model with a ZLB on interest rates and stochastic volatility under optimal discretionary monetary policy.

temporary risk shocks have to be larger to generate significant trade-offs for monetary policy.

10. Conclusion

In the canonical New Keynesian model, expectations are negatively skewed when risk is high relative to the available monetary policy space. Inflation settles materially below target in the absence of disturbances under optimal discretionary policy. Changes in the perception of risk give rise to cost-push effects regardless of the source of risk. The model is too simple to assign any great significance to quantitative results, and dynamic responses are likely to be too immediate in the purely forward-looking framework. But the results are indicative of the direction of the effects of risk in actual economies operating in an environment in which agents have reason to worry that monetary policy may be constrained in the foreseeable future. The new normal may be one in which monetary policy should lean against a re-anchoring of inflation expectations below target by operating the economy above potential in normal times. The analysis further points toward a monetary policy strategy aiming for an inflation rate *above*, but close to, 2 percent. The results also suggest that monetary policymakers should respond nimbly to changes in the perception of risk even as the economy escapes the ZLB.

Appendix A. Optimality Conditions

The policymaker minimizes (7) subject to (1), (2), and (3), taking expectations as given so that $E_t x_{t+1} = \bar{x}^e_{t,t+1}$ and $E_t \pi_{t+1} = \bar{\pi}^e_{t,t+1}$. To solve the optimal monetary policy problem, form the Lagrangian

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\left(\pi_t^2 + \kappa x_t^2\right) + \mu_{\pi,t}\left[\pi_t - \beta\bar{\pi}^e_{t,t+1} - \kappa x_t - u_t\right] \\ &+ \mu_{x,t}\left[x_t - \bar{x}^e_{t,t+1} + \frac{1}{\varsigma}\left(i_t - E_t\pi_{t+1} - r_t^*\right)\right] - \mu_{i,t}\left(i_t + i^*\right),\end{aligned}$$

where μ_π, μ_x, and μ_i are the multipliers on the three constraints. In addition to (1), (2), and (3), the Kuhn-Tucker conditions are

$$\begin{aligned}\pi_t + \mu_{\pi,t} &= 0 \\ \lambda x_t - \mu_{\pi,t}\kappa + \mu_{x,t} &= 0 \\ \mu_{x,t}\frac{1}{\varsigma} + \mu_{i,t} &= 0 \\ \left(i_t + i^*\right)\mu_{i,t} &= 0 \\ \mu_{i,t} &\geq 0.\end{aligned}$$

By substitution, these conditions can be reduced to

$$\left(i_t + i^*\right)\left(\lambda x_t + \kappa \pi_t\right) = 0 \tag{A.1}$$

$$\lambda x_t + \kappa \pi_t \leq 0. \tag{A.2}$$

There are two cases for condition (A.1) to consider.

Case 1. *If* $\lambda x_t + \kappa \pi_t = 0$*, (A.2) also holds, and the solution is determined by (1), (2), and (8). The Kuhn-Tucker conditions are all satisfied in this case if and only if* $i_t + i^* \geq 0$*. If it happens that* $i_t + i^* = 0$*, the ZLB is just binding.*

Case 2. *If* $i_t + i^* = 0$*, condition (3) holds, and dynamics are determined by (1) and (2) with* $i_t = -i^*$*. Now, we must have* $\lambda x_t + \kappa \pi_t \leq 0$ *for the Kuhn-Tucker conditions to be satisfied. If it happens that* $\lambda x_t + \kappa \pi_t = 0$*, the ZLB is just binding.*

If there exists an $i_t = i_1 > -i^*$ such that (1), (2), and (8) holds for some values $x_t = x_1$ and $\pi_t = \pi_1$, then setting $i_t + i^* = 0$ would imply that $x_t > x_1$ and $\pi_t > \pi_1$ by (1) and (2) so that $\kappa\pi_t + \lambda x_t > 0$. Hence, if the Kuhn-Tucker conditions are satisfied in case 1 given realizations of $\bar{x}^e_{t,t+1}$, $\bar{\pi}^e_{t,t+1}$, u_t, and r^*_t, case 2 will not be a candidate for a solution.

In sum, the policymaker chooses the unconstrained optimal policy allocation characterized by (1), (2), and (8) whenever it is feasible, and sets $i_t + i^* = 0$ when it is not. In the latter case, monetary policy will be insufficiently stimulatory in the sense that (A.2) holds with inequality rather than equality as in (8).

Appendix B. Solution

In each state (ϵ, u) in the $n_\epsilon \times n_u$ state space in period t, expectations are taken as given so that $E_t x_{t+1} = \bar{x}^e_{t,t+1}(\epsilon, u)$ and $E_t \pi_{t+1} = \bar{\pi}^e_{t,t+1}(\epsilon, u)$. Combining (1) and (8) in the form

$$\pi_t(\epsilon, u) = \beta \bar{\pi}^e_{t,t+1}(\epsilon, u) + \kappa x_t(\epsilon, u) + u_t(\epsilon, u)$$

$$\pi_t(\epsilon, u) = -\frac{\lambda}{\kappa} x_t(\epsilon, u)$$

gives the unconstrained optimal allocation

$$\pi_t^{opt}(\epsilon,u) = \frac{\lambda}{\lambda+\kappa^2}\left[\beta\bar{\pi}^e_{t,t+1}(\epsilon,u)+u_t(\epsilon,u)\right]$$
$$x_t^{opt}(\epsilon,u) = -\frac{\kappa}{\lambda+\kappa^2}\left[\beta\bar{\pi}^e_{t,t+1}(\epsilon,u)+u_t(\epsilon,u)\right].$$

The interest rate consistent with this allocation follows from (2):

$$i_t^{opt}(\epsilon,u) = \bar{\pi}^e_{t,t+1}(\epsilon,u) + r_t^*(\epsilon,u) - \sigma\left[x_t^{opt}(\epsilon,u) - \bar{x}^e_{t,t+1}(\epsilon,u)\right].$$

If $i_t^{opt}(\epsilon,u) \geq -i^*$, $\left\{x_t^{opt}(\epsilon,u), \pi_t^{opt}(\epsilon,u), i_t^{opt}(\epsilon,u)\right\}$ is the solution for state (ϵ,u) in period t. If the ZLB is binding so that $i_t^{opt}(\epsilon,u) < -i^*$, the interest rate is set to $i_t^{zlb}(\epsilon,u) = -i^*$. Now from (2) and (1):

$$x_t^{zlb}(\epsilon,u) = \bar{x}^e_{t,t+1}(\epsilon,u) - \frac{1}{\sigma}\left[-i^* - \bar{\pi}^e_{t,t+1}(\epsilon,u) - r_t^*(\epsilon,u)\right]$$
$$\pi_t^{zlb}(\epsilon,u) = \beta\bar{\pi}^e_{t,t+1}(\epsilon,u) + \kappa x_t^{zlb}(\epsilon,u) + u_t(\epsilon,u).$$

Hence, the solution for $y_t(\epsilon,u) \in \{x_t(\epsilon,u), \pi_t(\epsilon,u), i_t(\epsilon,u)\}$ for all nodes (ϵ,u) in the state grid is

$$y_t^{sol}(\epsilon,u) = \begin{cases} y_t^{opt}(\epsilon,u) & \text{if } i_t^{opt}(\epsilon,u) \geq -i^* \\ y_t^{zlb}(\epsilon,u) & \text{if } i_t^{opt}(\epsilon,u) < -i^*. \end{cases}$$

Ex ante expectations across the state grid can now be found as

$$\bar{\mathbf{x}}^e_{t-1,t} = \mathbf{P}_\epsilon \mathbf{x}_t^{sol}\mathbf{P}_u'$$
$$\bar{\pi}^e_{t-1,t} = \mathbf{P}_\epsilon \pi_t^{sol}\mathbf{P}_u',$$

where $\mathbf{P}_\epsilon$ and $\mathbf{P}_u$ are Markov transition matrices of dimensions $n_\epsilon \times n_\epsilon$ and $n_u \times n_u$, respectively.

The solution algorithm iterates the solution backwards from some period $t = T \gg 0$ to $t = 0$, initiated with $\bar{x}^e_{T,T+1} = \bar{\pi}^e_{T,T+1} = \mathbf{0}$. The state-contingent model solution is then $\left\{x_0^{sol}(\epsilon,u), \pi_0^{sol}(\epsilon,u), i_0^{sol}(\epsilon,u)\right\}$.

Appendix C. Calculation of Model Statistics

The stochastic steady state of model variable $y_t \in \{i_t, \pi_t, x_t\}$ is simply

$$y \equiv y_0^{sol}(\epsilon = 0, u = 0).$$

To calculate unconditional expectations, note that for each shock process $z_t \in \{\epsilon_t, u_t\}$ with Markov transition matrix $\mathbf{P}_z$, the stationary unconditional distribution, $\mathbf{d}_z$, satisfies $\mathbf{d}'_z = \mathbf{d}'_z\mathbf{P}_z$, or equivalently $(\mathbf{I} - \mathbf{P}'_z)\mathbf{d}_z = 0$. Hence, the unconditional distribution can be found as the normalized eigenvector associated with the unitary eigenvalue of $\mathbf{P}'_z$. The unconditional probability distribution over the state space is then the $n_\epsilon \times n_u$ vector product $\mathbf{D} = \mathbf{d}_\epsilon\mathbf{d}'_u$. The unconditional expectation of model variable y_t is found as

$$E(y) \equiv \sum_\epsilon \sum_u D(\epsilon, u) y_0^{sol}(\epsilon, u).$$

The unconditional variance is

$$\sigma^2(y) \equiv \sum_\epsilon \sum_u D(\epsilon, u) \left[y_0(\epsilon, u) - E(y)\right]^2.$$

Appendix D. Solution with Stochastic Volatility

With stochastic volatility, the discrete state space is extended to the $n_\epsilon \times n_u \times n_\sigma$ grid (ϵ, u, σ). Each period, the risk shock is assumed to be drawn first, followed by the level shocks given the realization of risk. With this timing assumption, a conventional method can be used to approximate the risk shock process (6) for a given volatility of $\nu_{\sigma,t}$, resulting in a $n_\sigma \times 1$ state space and an associated Markov transition matrix $\mathbf{P}_\sigma$. Let the set of grid points be $\tilde{\sigma} = \{\tilde{\sigma}_1, \tilde{\sigma}_2, \ldots, \tilde{\sigma}_{n_\sigma}\}$, centered at $\tilde{\sigma}_i = \bar{\sigma}$ for $i = n_\sigma - (n_\sigma - 1)/2$. The $n_z \times 1$ state space for each $z_t \in \{\epsilon_t, u_t\}$ can then be selected following the approach in Tauchen (1986) for the underlying level of risk, $\bar{\sigma}$. Assuming $\varsigma = 1$, the maximum value in the grid is set to

$$z_{n_z} = m \left(\frac{\bar{\sigma}^2}{1 - \mu_z}\right)^{\frac{1}{2}}$$

for some $m \in Z^+$, the minimum value to $z_1 = -z_{n_z}$, and the distance between states to $w = (z_{n_z} - z_1)/(n_z - 1)$. While the state space itself is kept fixed, transition probabilities now depend on the realization of risk. Each element in each Markov transition matrix $\mathbf{P}_z^\sigma = (p_{z,jk}^\sigma)_{n_z \times n_z}$ for each state of σ_t can be found using Tauchen's (1986) formula

$$p_{z,jk}^\sigma = \begin{cases} F\left(\frac{z_k - \mu_z z_j + w/2}{\sigma}\right) & \text{if } k = 1 \\ F\left(\frac{z_k - \mu_z z_j + w/2}{\sigma}\right) & \\ \quad -F\left(\frac{z_k - \mu_z z_j - w/2}{\sigma}\right) & \text{if } k \in \{2, 3, \ldots, n_z - 1\} \\ 1 - F\left(\frac{z_{n_z} - \mu_z z_j - w/2}{\sigma}\right) & \text{if } k = n_z, \end{cases}$$

where $F(.)$ is the standard normal cumulative probability distribution.

Expectations are taken as given in each state in period t so that $E_t y_{t+1} = \bar{y}_{t,t+1}^e(\epsilon, u, \sigma)$. The state-contingent solution $y_t^{sol}(\epsilon, u, \sigma)$ conditional on these expectations as well as the ex ante expectations conditional on the level of risk, $\bar{\mathbf{y}}_{t-1,t}^{e|\sigma}$, can be found as outlined in appendix B. But the calculation of the unconditional ex ante expectations, $\bar{y}_{t-1,t}^e(\epsilon, u, \sigma)$, requires an additional step with stochastic volatility. With a slight abuse of notation, these expectations can be found as

$$\bar{y}_{t-1,t}^e(\epsilon, u, \sigma = \tilde{\sigma}_i) = \sum_j p_{\sigma,ij}\, \bar{y}_{t-1,t}^{e|\sigma_j}(\epsilon, u),$$

where $p_{\sigma,ij}$ denotes the ij-th element of $\mathbf{P}_\sigma$.

The state-contingent model solution with stochastic volatility can now be found by iterating the period-t solution, $y_t^{sol}(\epsilon, u, \sigma)$, backwards from some period $t = T \gg 0$ to $t = 0$, initiated with $\bar{x}_{T,T+1}^e(\epsilon, u, \sigma) = \bar{\pi}_{T,T+1}^e(\epsilon, u, \sigma) = 0$, and calculating $\bar{x}_{t-1,t}^e(\epsilon, u, \sigma)$ and $\bar{\pi}_{t-1,t}^e(\epsilon, u, \sigma)$ in subsequent iterations, until convergence.

Model statistics for the case of stochastic volatility are calculated using straightforward extensions of the formulas in appendix C to three dimensions with $D(\epsilon, u, \sigma) = d_\sigma(\sigma) D(\epsilon, u \mid \sigma)$.

Appendix E. Risk Shocks with Stochastic Volatility

Following Koop, Pesaran, and Potter (1996), the generalized impulse response function for the risk shock process is defined as

$$\begin{aligned} GI^y_{t+n} &= E_t(y_{t+n} \mid \sigma_t = \tilde{\sigma}_i, \epsilon_t = 0, \ldots, \epsilon_{t+n} = 0, \\ &\qquad u_t = 0, \ldots, u_{t+n} = 0) \\ &\quad - E_t(y_{t+n} \mid \sigma_t = \bar{\sigma}, \epsilon_t = 0, \ldots, \epsilon_{t+n} = 0, u_t = 0, \ldots, u_{t+n} = 0) \end{aligned}$$

when $\tilde{\sigma}_i \in \tilde{\sigma}$ is the impact value of risk following the period-t innovation. This definition maintains the assumption that risk never materializes in nonzero-level disturbances along the adjustment path. By the Markov chain approximation we have

$$\begin{aligned} GI^y_{t+n} = &\sum_j p^n_{\sigma,ij}\, y^{sol}_0(\epsilon = 0, u = 0, \sigma = \tilde{\sigma}_j) \\ &- \sum_j p^n_{\sigma,mj}\, y^{sol}_0(\epsilon = 0, u = 0, \sigma = \tilde{\sigma}_j), \end{aligned}$$

where $m = n_\sigma - (n_\sigma - 1)/2$ and $p^n_{\sigma,ij}$ denotes the ij-th element of $\mathbf{P}^n_\sigma$ (i.e., the n-th power of the transition matrix $\mathbf{P}_\sigma$).

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